Traditional: 3-03
Themed: 10-04

Projectile Motion

Resolving Vectors Mathematically &
Projectiles at Angles

Vectors & Projectiles definitions

- **Projectile Motion** – Two-dimensional motion only affected by gravity (zero air resistance is assumed)
- **Resolve** – Process of finding the X and Y components of a vector
- **Resultant** – The sum of two or more vectors
- **Horizon** – Line where the earth appears to meet the sky
- **Ground speed/velocity** – motion of an object relative to the ground
- **Air speed/velocity** – motion of an object relative to the air it's moving in
- **Wind speed/velocity** – motion of air relative to the ground
- **Range** – The horizontal distance a projectile travels (Δx)

Solving Projectile Motion Problems

- X and Y motion are at right angles (independent of each other)
- Time (t) is the variable they have in common
- Nifty works the same as before (four variable, you must know 3, sad face our same friend)
- One X direction equation (must know two to use)
- Vx doesn't change, Vy does (that why there's Vi and Vf)
- Symmetry assumptions (same time up as down, final speed = initial speed, same distance up as distance down)

Y direction (where velocity is changing)

Equation	a	*t*	v_i	v_f	Δy
$v_f = v_i + at$	√	√	√	√	☹
$\Delta y = \frac{(v_i + v_f)}{2} t$	☹	√	√	√	√
$\Delta y = v_i t + \frac{1}{2} a t^2$	√	√	√	☹	√
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√

X direction (where velocity is not changing): $\Delta x = v_x t$

Solving Projectile Motion Problems

- Two kinds of problems:
  - Horizontal motion problems
    - Vi = 0
    - Shoots a gun horizontally, rolls off of a table, etc.
  - Projectiles shot at angles….hard!
- Two part problems:
  - Use X equation to find t, then use t as third Y direction known
  - Use Nifty to find t, then use t as second X direction known

Y direction (where velocity is changing)

Equation	a	t	v_i	v_f	Δy
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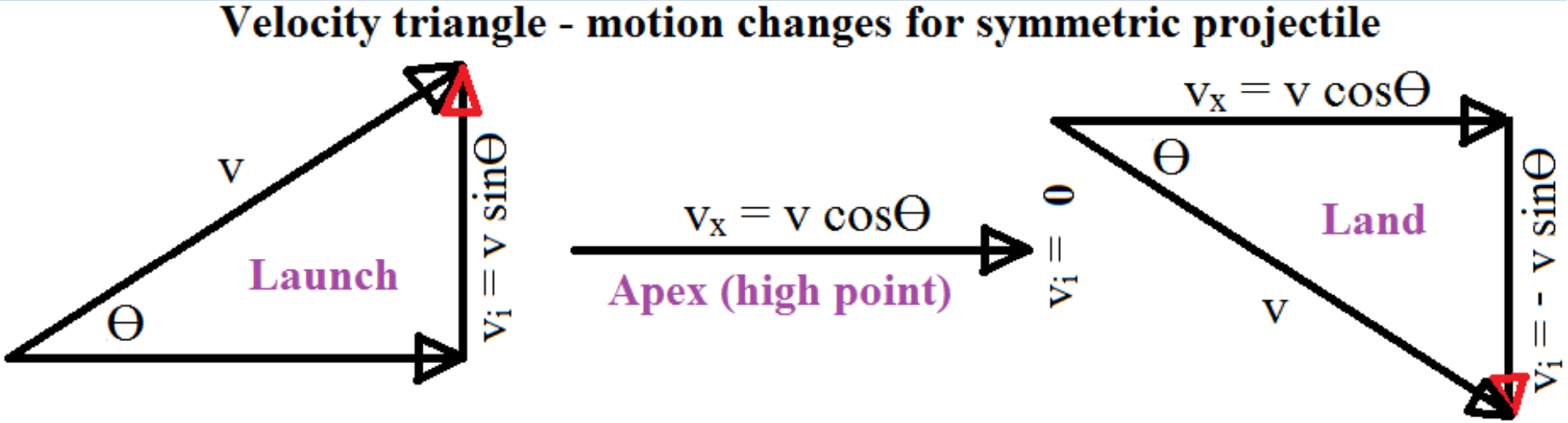
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√

X direction (where velocity is not changing): $\Delta x = v_x t$

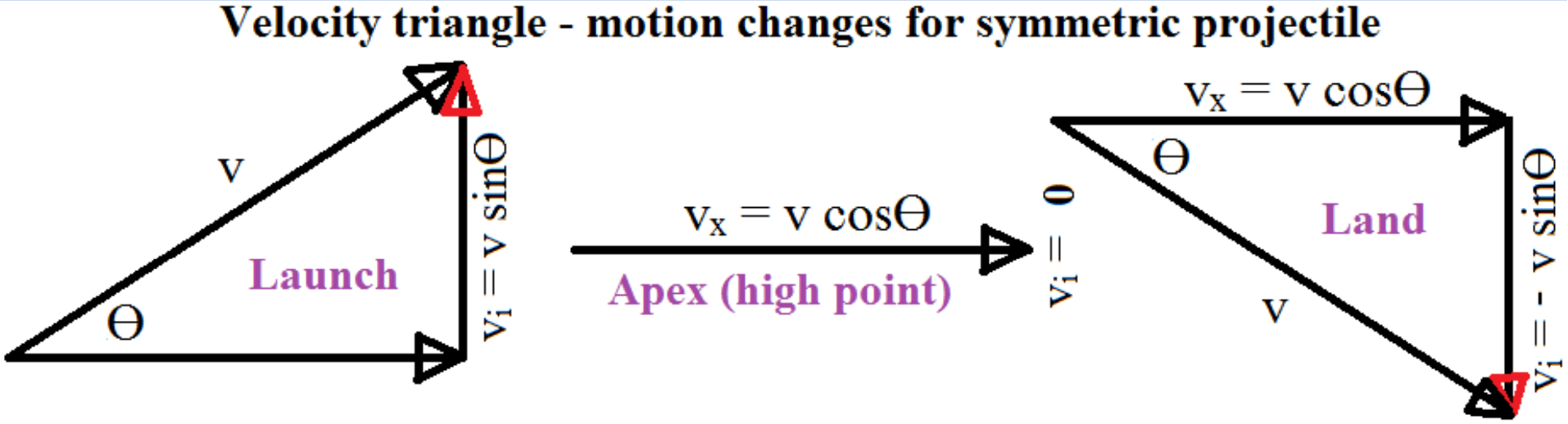
How motion changes during projectile's lifetime

- When is speed a maximum? A minimum?
- Is speed increasing with altitude gain? Why?
- If v>0, what angles produce zero range?
- What angle produces min and max hang time?
- What angle is"best" compromise between big Vx and big hang time producing biggest range?



More conclusions: Effects of angle variations

- Small angles = big Vx and make rapid X directions progress, but don't stay airborne long, so small range
- Big angles = big hang times, stay up a long time, but have tiny Vx, so small range
- Best range is equal compromise between hang time and big Vx (45° angle)
- Complementary angles have equal trade-offs, so have <u>equal</u> ranges
- See how Vy=0 at max height? Does the acceleration switch directions when that happens?



Effect of angle above horizon on…

- What angle(s)
  - Maximizes hang time?
  - Maximizes range?
  - Give equal ranges?
  - Maximizes peak height?
  - Maximizes Vx?
  - Makes a football go farthest?
  - Tennis players only: Is hardest for a tennis player to hit? (Think John Isner vs. Andy Roddick)

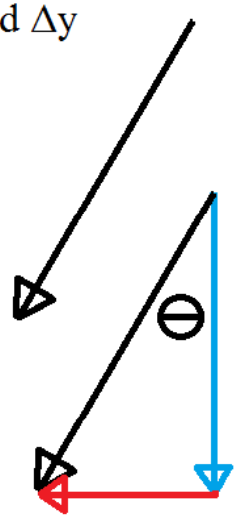
Resolving Vectors

- Finding the x and y components of a vector
- Useful to find Vi and Vx (why do you want Vi and Vx anyway?)
- For finding resultants (adding vectors)

Resolving Vectors

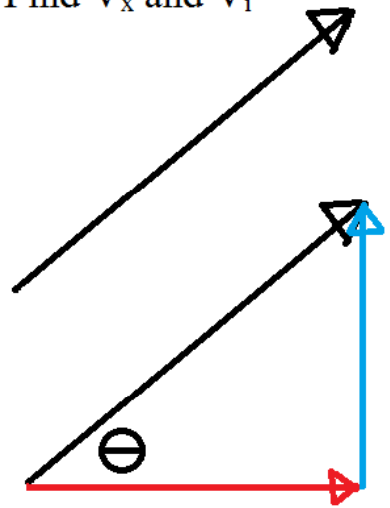
- Process:
  1. Sketch vector to be resolved
  2. Vector becomes hypotenuse, finish triangle (one side is totally x and the other side is totally y)
  3. Use trig to find values (careful!)

Displacement, R = 3.7 km 30° W of S
Find Δx and Δy



Can you do the trig to find Δx and Δy?

V = 30 m/s 41° N of E
Find V_x and V_i



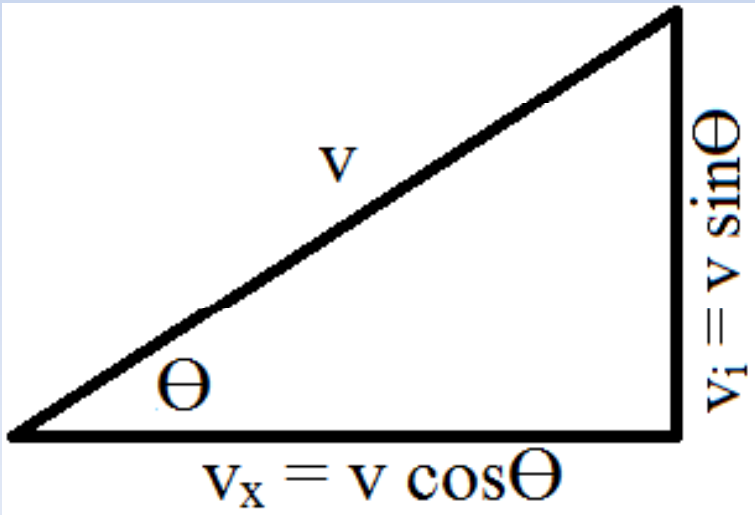
Can you do the trig to find V_x and V_i?

Solving Projectiles At Angles

- What the heck does this even mean?
- Examples:
  - Tom throws a ball 30 m/s at an angle of 20° above horizontal from an initial height of 1.4 m. How far will the ball go when striking the ground?
  - What minimum speed must a pass be thrown to reach a receiver 30 m downfield when caught at the same height as thrown? (YES, this implies a specific angle!)
- We're not solving these *yet*!

Solving Projectiles At Angles

- Find as many of X/Y motion variables as possible
  - If you have two horizontal knowns, you can find 3rd
  - If you have three vertical knowns, you can find other 2
- If possible, RESOLVE the velocity vector in the x-direction and y-direction components
- Takes place on earth?
- ΔY is negative if down, 0 if same level
- Time ties X and Y together
- What equation can I use?
- HARD: Algebra substitution



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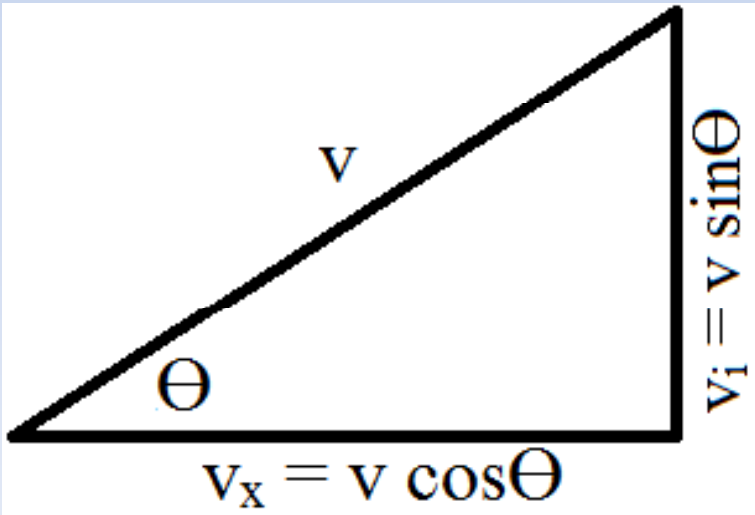

Solving Projectiles At Angles

- Wanna try these now? (*harder than horizontal*)
  - Tom throws a ball 30 m/s at an angle of 20° above horizontal from an initial height of 1.4 m. How far will the ball go when striking the ground?...*More than one way to do it, I'd avoid quadratic*
  - What minimum speed must a pass be thrown to reach a receiver 30 m downfield when caught at the same height as thrown? (YES, this implies a specific angle!)…*hint: solve for t in terms of V using X-stuff*