4. G-forces and Turns

Newton's Laws & Circular Motion

Goals of this unit:

1. Understand the nature of the following forces: weight, normal, friction, centripetal force
2. Use free body diagrams to determine the g-forces at different points on a coaster ride
3. Explain the motion of a coaster using Newton’s three laws

1.0 Introduction: how many g's can a rider take?

During lift off a Space Shuttle astronaut feels up to 3 g’s. Fighter pilots wear special “g-suites” to help them pull up to 7 g’s in high speed turns. Parachute paratroopers fall at zero g’s. On a typical coaster, g forces can be as low as –3 and as high as +5. Race car drivers experience intense g’s if they suddenly accelerate or decelerate. But what is a g-force? The letter “g” refers to the force of gravity you feel on your body. When you sit on a chair, you feel a force of 1 g. You push down with a force equal to your weight, and the seat pushes up with an equal force of 1 g. Riding up on an express elevator, you might feel as if you are twice as heavy if the floor pushes up on your feet with a force of 2 g’s. When pilots use their ejection seats to escape, they feel up to 20 g’s for a few seconds!

The human body has definite limits regarding g-forces. The higher the g-force, the heavier is each ounce of blood and therefore that much harder for the heart to pump the blood. If not enough blood gets to the brain, a person may black out. Riders can easily survive a few seconds at less than 5 g’s. A few seconds at higher g’s can lead to tunnel vision, hemorrhaging, unconsciousness (black out), and even death. Any exposure to 40 g’s is instant death☹. Many high g rides are designed so the rider’s head is positioned lower than normal and the knees slightly raised to help prevent black out. A few seconds between 0 g and 1 g can cause queasiness and dizziness, but no permanent medical problems. The least force on a rider is at 0 g. Negative g's are also possible, and are similar to positive g's except blood is pushed upwards into the head instead of pulled away. At negative g forces , blood becomes lighter so that the heart actually pumps too much blood to the brain causing red out. A few seconds of red out may lead to recoverable migraine headaches and tissue swelling. But at forces much less than –3g’s, blood vessels will burst and can cause instant death.. Very few rides have negative g forces and these experiences are designed to last less than a second. The physiological affects of experiencing g-forces for several seconds are tabulated below:

g < -3	Death due to massive blood pressure & leakage
g = -3 to -1	Severe headaches, swelling
g = 0	Weightless feeling, dizziness, queasiness
g = 0 to 1	Partial weightlessness, less intense sensations
g = 1 to 5	Squeezed feeling, light headedness
g > 6	Black out, tunnel vision, hemorrhaging
g > 40	Instant death due to lack of blood pressure

The goals of this unit are to understand the forces involved in coasters, to know how these forces affect a coaster's motion, and to determine the g-force on a rider anywhere along a coaster ride. A great physicist named Isaac Newton discovered three laws of motion which will guide us to meeting these goals.

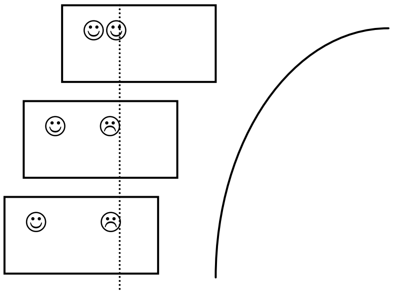
2.0 Forces & Newton's Three Laws

A **force** is a push or a pull. People exert forces all the time. A student pushes the door open, pulls on a shoestring, pushes down on the floor to jump in the air, or pulls a book from a locker. Did you know forces are exerted by inanimate objects? The Earth pulls on you all the time with a force of gravity called **weight**. The floor pushes up on us all the time to hold up our weight. That force of support is called the **normal** force. **Friction** is a force of resistance between two objects moving against another. **Air resistance** is a special force of friction since one of the objects is air. **Tension** is a force along taunt ropes. Clearly many forces affecting our lives are from non-living things. Newton discovered three laws of physics that explain how forces change the motion of objects.

2.1 Newton's 1st law

> "An object will continue to stay at rest or move at constant speed in a straight line unless there is a net force acting on it"

Net force is the resulting vector sum of all acting forces which may be in different directions. For example, the net force in lifting a book is your lifting force minus the weight of the book. Newton's first law explains what happens in the absence of a net force. Nothing new! In street language, things left alone don't change. The physics term for resistance to change is called **inertia**. Inertia is most similar to the concept of mass, but there is no equation for inertia. A large truck stopped at a red light has a lot of inertia. The truck wants to stay at rest, and requires a large net force to move it. A large truck speeding down the highway also has much inertia. The truck wants to keep speeding and would require a great net force to slow it down, or to make it turn. Objects already moving in a straight line tend to continue to move in a straight line. Back in the days of bucket car seats, it was common to get your date to sit closer to you by using "**benefit corners**". The driver would simply turn hard to the right, and hold onto the steering wheel. Meanwhile the passenger would continue with (her) inertia to move in a straight line as the car moved to the right. In other words, the passenger would slide over to the left closer to the driver for hopefully a mutually beneficial corner. . To see how this is done, study the sequenced diagrams below:



2.2 Newton's 2^{nd} law

Newton's second law is the most famous:

The acceleration of an object is equal to the net force divided by its mass.

$$a = Fnet / m \quad \text{or} \quad Fnet = ma$$

This law explains states that the same force applied to a small object will create more acceleration than when applied to a more massive object. This is the reason racecars use the most forceful engine with the lightest mass. When the force is only gravity, then the acceleration is just g , 9.8 m/s^2 and the net force is just weight. Roller coasters are also called thrill machines since the forces on you and hence your acceleration can change quite rapidly and unpredictably. The acceleration during a coaster ride can vary from negative g's, to zero g, or to many positive g's all during one short ride. This is possible since both the size and direction of the forces may be changing. We will explore these possibilities later in section 4.

The net force can cause an acceleration that is a change either in speed or direction. When there is a simple change in speed and not direction, the acceleration can be found from the nifty equations of motion:

Acceleration as a change in speed:

$$V_f = V_i + a\,t$$
$$V_f^2 = V_i^2 + 2ad$$
$$d = V_it + 1/2at^2$$
$$F_{net} = ma$$

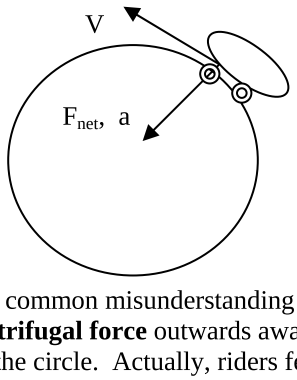
The direction of the acceleration is always the same as the direction of the net force. For example, if the track exerts a normal force upwards greater than the weight, a rider will be accelerated upwards. When braking, a large net force is exerted backwards on the coaster causing a negative acceleration, i.e. deceleration. What does it mean when the net force is zero? There are two possibilities, and both create zero acceleration. First, there are no forces on the coaster. The speed is zero. Second, there are forces but they cancel another by being equal but opposite direction. The speed is constant (and in a straight line).

When a net force causes a change in direction, that net force is also called the **centripetal force**. For motion in circles, turns, and loops there must be a centripetal force to make the coaster turn. The acceleration is not found using the nifty equations, but rather an equation that depends on the speed of the coaster and the radius of the turn:

Acceleration as a change in direction

$$a = v^2 / r$$
$$F_{net} = ma = mv^2 / r$$

The direction of both the centripetal force and acceleration is towards the center of the circle, while the velocity is tangent to the path:



There is a common misunderstanding that riders feel a **centrifugal force** outwards away from the center of the circle. Actually, riders feel the normal force of the track pushing against them to stop their inertia which would otherwise send the rider in a straight line

2.3 Newton's 3nd law

Forces always come in pairs. This oddity was first recognized by Newton in his 3rd law:

> Whenever one object exerts a force on second object, the second object exerts simultaneously on the first object a force equal in size and opposite in direction.

When you push down on the floor to jump, the floor also pushes up on you. You can feel the floor's uplifting force on your feet. When you lean on a wall, the wall pushes back on you. That is why after a while your body will feel tired. Forces come in pairs from non-living objects as well. The earth pulls you down, and you pull the earth up. The earth hardly moves since it is so massive, but it is true that you can move the earth!

One of the paired forces is called the action force, and the other is the reaction force. Like clapping hands, one cannot exist without the other. For example, consider why you can't easily walk on super slippery ice. You try with your feet to push on the ice, but the ice can't push back since there is not enough friction. Therefor, your action force on the ice cannot exist without the reaction force of the ice on you. Action and reaction forces can be easily identified by simply reversing the sentence describing a force. For example, when kicking a ball you would say the action force is me hitting the ball. The reaction force is the ball hitting me. How about a bullet shot from a gun? The gun pushed the bullet forward, and the bullet pushes the gun backwards.

The careful reader might think that action forces must be stronger than reaction forces to make anything move. This is a common misunderstanding. Consider a tug a war between two teams. The net force on a flag tied to the middle of the rope is the stronger team's pull minus the weaker team's pull. The two teams are not action-reaction pair forces since they do not share the same force. The stronger team pulls on the rope, and the rope pulls back with the same force. The rope and the stronger team are action-reaction force pairs. Their forces do cancel out. But that doesn't mean the flag can't move. The flag moves due only to forces on it- the two teams. The two separate reaction forces of the rope on the teams do not affect the flag but are necessary for the teams to exert their action forces in the first place. So now can you explain how it is possible to push a cart out of the mud if your force on the cart is equal to the cart's force on you? Sure, the car moves if my pushing force is greater than the friction of the mud. The reaction force of the cart is on me, not on the mud so it doesn't matter how much the cart pushes back on me as long as I push ahead on the cart more than the mud pushes back on the cart. Alas, Newton's laws even work in sticky situations!

2.4 Summary

It takes no additional force to continue staying at rest or moving in a straight line. It does take a net force to change speed or direction of speed. Net force is determined by adding all the forces which may be in different directions. The smaller the mass, the greater the change for the same net force. If there is a change in direction, the net force is called centripetal force. In the case of roller coasters, the track provides the centripetal force needed in turns and loops. Riders feel as if they are being pushed into their seats, when actually the seats are being pushed into them by the centripetal force. The rider's reaction force is to push back on their seats and this is the force they feel, not some imaginary centrifugal force. Every force is either a push or a pull and every force comes in pairs, an action force and a reaction force.

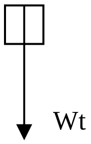
3.0 Free body diagrams of forces

There are many forces involved in rides at theme parks. We need to know how to calculate each force and know each force's direction so that the net force and hence acceleration of a ride can be determined. In this way the feelings a rider experiences can be understood and as described later measured in terms of g-forces. A free body diagram is a simple way to show how many forces act on an object. The object (e.g. a coaster car) is drawn as a box, and forces on it are show as arrows. The length of the arrow indicates the size of the force, and the tip of the arrow points in the direction of the force. For example, an arrow 2 inches long pointed down could be the weight of the object, while another arrow 2 inch long pointing up could be the equal but opposite normal force of the floor holding up the object.

3.1 Weight (force of gravity)

Weight is the pulling force of gravity between an object and the Earth. Gravity is an attraction that exists actually between any two masses. There is a pull between you and the moon, and even between you and another person. The force of gravity is stronger for objects that are more massive and closer together. On Earth, it makes sense to consider only the strongest gravity – the one between us and Earth. Since the distance between you and the center of the Earth varies little while walking on this planet, the force is nearly constant and is found simply by multiplying your mass in kilograms by the number 9.8 m/s^2, which is the acceleration due to gravity. The unit for weight is Newtons (N) as is true for all forces.

Weight (Wt) = $m*g$ where $g = 9.8\ m/s^2$

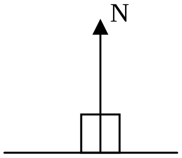


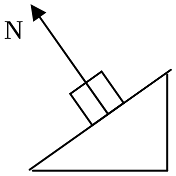
The direction of weight is always vertical and pointing down since the force of the Earth's gravity pulls us down towards the center of the Earth. A popular song by Richard Petty suggests free falling is very common. The term "**free fall**" refers to an object that is falling only under the influence of gravity. Other forces like air resistance do not exist or can be neglected if an object is free falling. Objects under free fall speed up at a rate of 9.8 m/s (about 22 mph) every second. So if roller coaster falling down a hill reaches 66 mph in 3 seconds, then it is truly free falling. Most likely, there is some air resistance and friction slowing the coaster.

It is also difficult to be actually **weightless**, that is having no weight. The farther you are from Earth, the lower is your weight. Orbiting Earth in the space shuttle, your weight is about 95% of that on Earth. Even on the moon, your weight is still about 1/6 that on Earth. To be truly weightless, you have to be an infinite distance away from Earth, or even more bizarre at exactly the center of the Earth! It is however easy to feel as if you have no weight. But to understand how, you must understand what is a normal force.

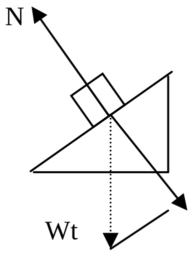
3.2 Normal Force

When standing on a floor, what holds you up? A force that supports an object is called the **normal force**. It is measured in Newtons and does not have an equation. Normal means at right angles; the normal force pushes up at right angles to a support surface. Hence, when you stand on a floor the normal force is pointed at the ceiling. If you stand on a ramp, then the normal force is pointed upwards perpendicular to the ramp:



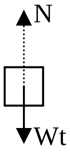


The normal force increases or decreases depending on the situation. When standing on a flat floor, the normal force must equal your full weight. Otherwise if the normal force was larger than your weight you would lift off the floor, or if smaller than you weight you would break through the floor. On a ramp, the normal force is less than your weight since the normal force only has to support that part of the weight pointing at an angle down (solid line below)and not the total weight pointing vertically down (dashed line):

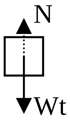


An even more common experience with changing normal forces occurs on elevators not moving at constant speed. Going up, the normal force must be greater than the weight to produce an upward acceleration. Going down, the normal force must be less than the weight to allow a controlled deceleration. The size of the normal force can be found from using Newton's second law as follows:

Going up: $F_{net} = N - Wt$
use $F_{net} = ma$ and $Wt = mg$
So $N = ma + mg$



Going down: $F_{net} = Wt - N$
Rearrange: $N = Wt - Fnet$
So $N = mg - ma$



The same elevator physics happens on a coaster but on a curved rise and fall. Near the bottom of each hill and the beginning of each loop, the track hooks up like the bottom of the letter "D" and pushes the coaster upwards with a normal force that exceeds the weight. Near the top of hills, bumps, and loops, the opposite effect happens. The track curves down and outward like the top of the letter "D" with a normal force smaller than the weight. The mathematical details will be examined later!

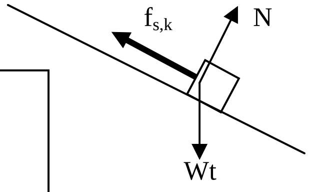
3.3 Friction

Friction is the force of resistance between two objects in contact with another. Friction points in the opposite direction of motion, parallel to the surfaces, and is measured in Newtons. As one might expect, friction depends on the tendency of the objects to resist another's motion, and how hard the surfaces are pushed against another. The fun equation for friction is given by multiplying the **coefficient of friction** (μ) times the normal force (N).

FRICTION

$f_{s,k} = \mu N$

where s is for static (not moving)
k is for kinetic (moving)
μ is the coefficient of friction



The greater the normal force, the greater the force pushing the surfaces together. The greater the **coefficient of friction**, the greater the tendency for the surfaces to bump or stick. The coefficient of friction is determined experimentally, and depends in part on the roughness of each surface and the chemical attraction between the two materials. Friction tends to be higher on wooden coasters than steel ones. .Friction will always point along the direction of the track and in the opposite direction of motion. In many situations, friction can be assumed negligible. However, friction is important since it constantly slows the coaster and eventually friction is used to stop the coaster. Surprisingly there are two types of friction, kinetic (meaning moving) friction (symbol is MF or $f_{,k}$) and static friction (SF or f_s). Static friction refers to the larger initial force that must be overcome to start rubbing one surface over another. Moving friction is the continued resistance felt as two surfaces move and rub over another. For coaster problems, static friction is only used when first starting a coaster. Otherwise the coaster is moving and friction is the moving friction type.

3.4 Air resistance

A type of friction, **air resistance** (symbol R) is a resistive force and points in the opposite direction of motion. Air resistance is rather complicated but in general increases with speed and the size of the moving object. Since roller coasters are rather aerodynamic and slow vehicles, we will neglect air resistance☺. Of course, it should not be neglected for rides that involve parachutes, sails, or light weight vehicles.

3.5 G-forces

On a roller coaster there are many "benefit corners, dips, and loops" in which your inertia sends you straight, but the track keeps the car turning. There is a common misconception that some "centrifugal" force is pushing you into your seat. In reality, the normal force of the track is pushing on your seat so that the car can make the turn. What you feel is actually the ***reaction*** force of your body to that seat or normal force: the seat pushes me and ***I push on the seat***. The harder you push back on the seat, the heavier you feel. This feeling is measured in g-forces. G-force is not actually a force, but a ratio of the reaction force you exert on your seat divided by your own weight. Your reaction force will be equal but in opposite direction to the normal force from the seat. Hence g-force can be found from the following equation:

$$\begin{aligned}\text{G-force} &= \text{your reaction force} / \text{your weight}\\ &= \text{Normal} / \text{weight}\\ &= N / mg \quad \text{where } g = 9.8\text{m/s}^2\end{aligned}$$

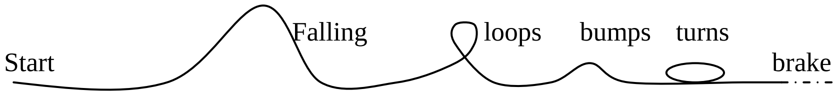
You many feel both vertical and horizontal g- forces depending on if you are pushing into the bottom or back of your seat respectively. We will only consider horizontal g-forces when the coaster starts and stops since this is when there are large horizontal accelerations and decelerations. Otherwise we will evaluate g-forces that are vertical in the sense they are perpendicular to the track.

Both positive and negative g's can be felt along a typical coaster ride. The main difference between positive and negative g's is the direction of the normal force. At positive g's, the normal force from the seat pushes on the rider, and the rider feels compressed into the back or bottom of the seat. The rider feels light headed or may black out as blood in the rider is pushed away from the brain. At negative g's, the seat is pulled more away from the rider, and there is a feeling of being bumped out of the seat. The blood's own inertia forces more blood to the head causing red out symptoms. The physiological implications on the rider are tabulated below:

g-force value	**Implications**
g-force > 1	Feel heavier than usual as if being squeezed
g-force = 1	Feel own weight, same as when not moving
g-force 0.5	Feel half your weight, as if may float above seat
g-force = 0	Feel weightless, as if actually floating above seat
g-force <1	Feel weightless and seat pulled away from you

4.0 Determining G-forces

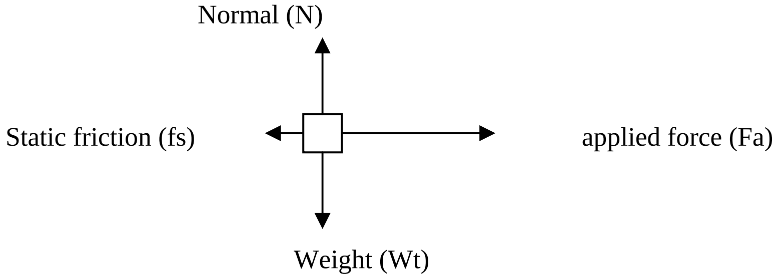
Here's our question: How many g-forces do you feel on your back and on your buttocks?
We will answer this question at 6 distinct spots on a coaster. Which spot would you guess has the highest g? The smallest g? The most negative g? Where would you feel weightless? Read on!



4.1 Starting

All coasters are started from rest with a push, which may be gentle or intense. We will assume the coaster starts on a level horizontal track and the seats have a bottom and back cushion.

Step 1: determine which forces are acting on you and draw a free body diagram.
Weight is always present and should be drawn first using an arrow that points down. Next consider if a normal force exists. Your weight is supported by the bottom of the seat, so there is a normal force pushing up. The size of the normal force will be exactly equal to your weight since there is no net acceleration in the vertical direction. However you do feel a horizontal acceleration due to the force of the initial push. We will call this initial push the **applied force** (Fa). There is also static friction resisting this push, but it must be smaller than the applied force for the coaster to start moving. We will ignore air resistance since the coaster is barely moving. The free body diagram for starting a coaster looks like:



Step 2: find the net force in both the horizontal and vertical directions and set equal to m*a

Horizontally (x-direction): $F_x net = ma_x = Fa - fs$

Vertically (y-direction): $F_y net = ma_y = N – Wt$

Step 3: calculate the g-forces using the normal force

Vertical (perpendicular to track)

In the vertical direction, the g- force depends on the normal and weight forces. Since there is no acceleration perpendicular to the track, the normal force must be exactly equal in size to the weight:

by definition: g-force = N / Wt

But $a_y=0$ so N=Wt so g-force = Wt/ Wt
g-force = 1

horizontal (parallel to track)

There is a horizontal acceleration since the coaster starts to move from rest. The applied force must be larger than the static friction force to produce a positive net force equal to ma, that is mass times acceleration. The rider feels the net force pushing him forward, so the normal force is equal to the net force. As shown below, the horizontal g-force can be most simply expressed in this case by the acceleration:

g-force = N / Wt
= Fnet / Wt
= ma / mg
= a/g

For gently starting coasters, you typically feel a small force on your back about half of your weight, or a g-force of 0.5. This corresponds to an initial acceleration of about half of what gravity accelerates objects.

Helpful hint: use nifty equations to find acceleration if given final speed, time, or distance!

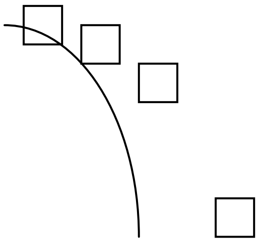
$V_f = V_i + a t$ $V_f^2 = V_i^2 + 2ad$ $d = V_i t + 1/2at^2$

4.2 falling down hills

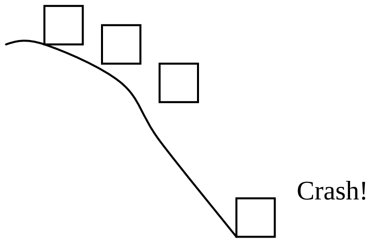
Many riders enjoy the queasy sense of weightlessness felt as a coaster begins to plummet down a steep hill. Indeed, the g-force must be zero. You feel as if you have no weight for the same reason that you feel no weight when falling through the air- there is no support! The tracks are curved down just right so that the coaster will fall with the wheels just barely above the tracks. There is no normal force or kinetic frictionl, only perhaps a small amount of air resistance. The curvature of the track is the same as that of a projectile in motion (we'll neglect friction and air resistance). Remember the distance fallen vertically ($d = 1/2at^2$) is independent of the distance moved laterally ($x = v*t$). The faster the coaster moves at the top of the hill, the shallower should the track be designed for accomplish 0 g's.

Consider what happens if the track is make too steep or too flat:
(the boxes show the natural curved projectile path of a falling coaster)

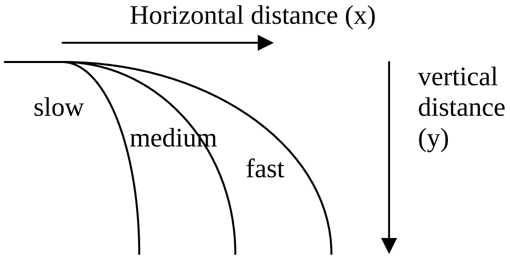
Too steep – coaster flies off



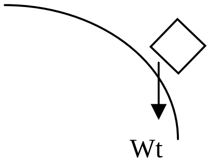
Too flat – coaster bounces off & crashes



Here is the correct track curve for 3 speeds – slow, medium, fast:



If the tracks are designed correctly for the known speed at the top of the hill, the only force on the riders will be their weight, which cannot be felt since the track does not need to provide support. Without a support, the normal force is zero and hence the g-force is zero!



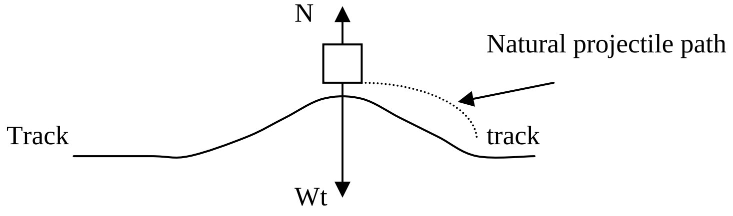
g-force = N / Wt

N = 0 since not touching so

g-force = 0

4.3 Bumps (small hills)

Riding over bumps (small hills) is similar to falling down the first hill except you are moving a lot faster when a coaster rides and falls over a bump. This increased speed means the coaster would like to fall over a less steep projectile path, but the track pulls it back. The result is the rider feels a negative g-force as his chair is pulled backwards from him. The riders enjoy a feeling of being bumped out of the seat, but coaster designers must make sure the force is not enough to flick passengers or cars into the air. Riding a bump requires a centripetal force since the car is traveling on the outside of a circle. As seen below in the free body diagram , the downward centripetal force provided by weight is effectively reduced by the normal force pointing up. This also means the normal force felt by the rider is reduced by the centripetal force:



Vertically: $F_{net} = Wt - N$

$ma = Wt - N$

so $\boxed{N = Wt - ma}$

then g-force = N / Wt

g-force = Wt - ma / Wt

so $\boxed{\text{g-force} = 1 - a/g}$

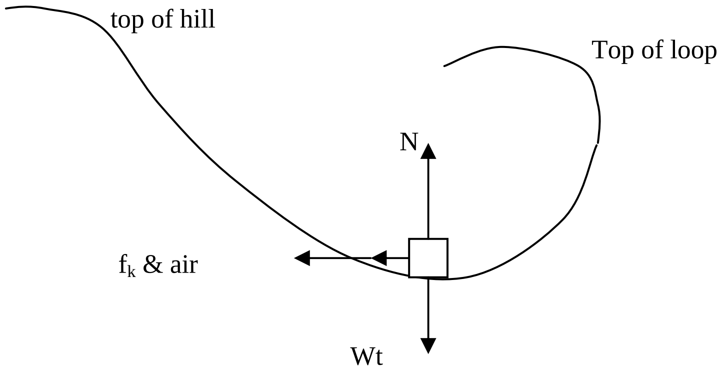
The centripetal acceleration a is given by v^2/ r where v is the coaster speed and r is the bump radius. If the speed is too high or the radius of the bump too small, then a/g will be larger than one and the g-force will be negative. In this case, the centripetal force is too large to be provided for by the weight and the coaster will lift off the tracks. Disaster is averted by designing small g-forces (less than –0.5) and using under-track wheels. Still at –0.5 g's, riders will need about half of their weight to hold themselves down in their seats. Seat belts will hold passengers from becoming air born, but any loosely held articles can literally be bumped out of the coaster. Likewise internal organs can be jarred causing some passengers discomfort, while others it brings great joy.

4.4 Loops

Loops (and turns) have the additional restriction that in the vertical direction the net force can't be zero or otherwise the coaster would not complete the loop. The two vertical forces are the normal force and weight. The direction of the normal force rotates going through the loop but is always pointed towards the center of the loop. Horizontally the only force is friction and air resistance both in a direction along the track length in the opposite direction of the coaster. Friction and air resistance are both small so that the rider feels typically a horizontal g-force near zero. The loop is actually not a perfect circle (to be explained why later!), so there will be some positive g-forces (typically 0.5 to 2 g's) on the rider's back near the top and bottom of the loop. Derivation of these forces requires trigonometry and will be not discussed. The vertical forces are far easier and more interesting! By vertical we will mean perpendicular to the track!

We will focus on the bottom, side, and top of the loop since here the forces here are at convenient angles. G-forces between these spot can be easily extrapolated once the "big picture" is seen.

Bottom of a hill or loop



The results we will discover for the bottom of the loop are the same for the bottom of a hill since the track is curving so as to push the coaster upwards above its natural path. There is no forward force. Newton's law of inertia states the coaster will continue to move in a straight line without any help. However, it takes a centripetal force to make the coaster turn. This force is provided by the normal force. At the bottom of the loop, the normal force must not only support the weight of the coaster, but also have some extra force left over to guide the coaster in a circle. The coaster seat will push up on the rider with a normal force larger than the rider's weight. Thus the rider will push back the seat with an equally large reaction force and hence a positive g-force much greater than one. The g-force can be determined once the normal force is extracted from the net force equation:

$$F_{net} = N-Wt$$
$$ma = N-Wt$$

So $\boxed{Wt + ma = N}$ then plug N into the g-force equation☺

Find g-force: $\text{g-force} = N / Wt$

$= (Wt + ma) / Wt$

$= (mg + ma) / mg$ use $Wt = mg$ so can cancel out all m's

$$\boxed{\text{g-force} = 1 + a/g}$$

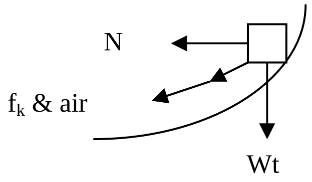
The greater the acceleration into the loop, the greater the g-force. Typical accelerations are several times greater than that of gravity (g= 9.8) so g-forces between 2.5 and 5 are commonly experienced. The acceleration is due to the centripetal force which alters the direction not the size of the speed.

The acceleration of a coaster moving along a circular path is found by the following relationship:

Centripetal acceleration: $\boxed{a = v^2 / r}$

where v is the speed of the coaster along the track and r is the radius of the track circle. The g-force will be much larger if the coaster is moving faster or moving along a tighter smaller circle. Anyone who has driven a car knows it takes more force to turn while moving fast, especially along hairpin-like turns.

Side of loop:



As the coaster goes up the loop, the normal force rotates to point toward the center and remains perpendicular to the tracks. As the coaster becomes more vertical, less of the weight is pushing down on the track and hence there is less support force needed. The normal force as well as the g-force decreases as the coaster approaches the mid-side of the loop. The normal force can't be zero here since it must still provide the same centripetal force (F = ma) to make the loop. In fact, the normal force is equal to the centripetal force. Once again we will consider only the g-force perpendicular to the tracks.

$F_{net} = N$ by looking at the free body diagram (normal is centripetal force)

$F_{net} = ma$

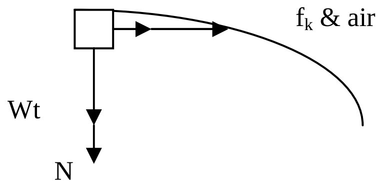
$\text{g-force} = N / Wt$

$= ma / mg$

$$\boxed{\text{g-force} = a / 9.8 \text{ m/s}^2}$$

Exactly half way up on either side of the loop, the weight is perpendicular to the normal force. The weight neither aids nor hinders the normal force. The normal force is the only centripetal force. The rider feels the seat push with this normal force, and the rider pushes back with an equal force. Hence the g-force only depends on the centripetal acceleration , i.e. the coaster speed and track radius.

Top of the loop:



As the coaster rises to the top, the normal force continues to decrease because above the midpoint the weight points more and more towards the center of the loop and thus helps provide a centripetal force. Once again, the g-force is determined from the normal force since that is the force riders react to.

$$Fnet = N + Wt$$
$$Fnet = ma$$

So $$\boxed{ma - Wt = N}$$

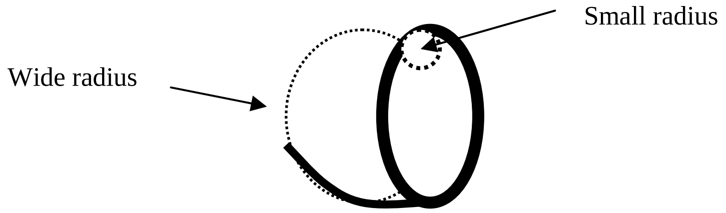
Then
$$\text{g-force} = N / Wt$$
$$= (ma - Wt) / Wt$$

$$\boxed{\text{g-force} = a/g - 1}$$

In the extreme case the normal force is zero at the top and the riders feel weightless for a moment. Typically loops are designed with a small normal force to ensure the coaster stays in its tracks.
A typical g-force is about 0.5 which means the riders feel about half their weight pressed into the seats. At this g-force, a/g equals 1.5 which means the minimal safe acceleration is about 1.5 times that of gravity.

The story of modern Klothoid loops:

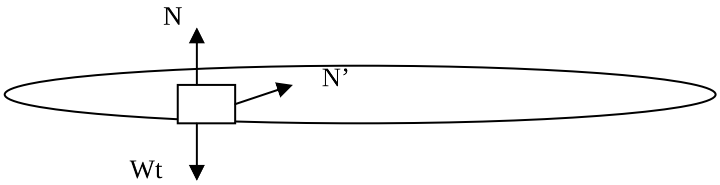
The careful reader will note that if a g-force of 0.5 at the top would correspond to a g-force of 2.5 at the bottom using an acceleration ratio a/g of 1.5. However a constant acceleration is not possible. As the coaster goes up the loop it looses speed and as it comes down it gains speed. If the acceleration is 1.5 g's at the top, it must be much larger at the bottom. It is possible to prove using conservation of energy to determine that the ratio a/g is 5 times bigger at the bottom than the top of the loop (the proof would make an excellent final exam problem don't you think?). A g-force of 0.5 at the top would then correspond to a g-force of 8.5. Most people feel uncomfortable above 3.5 g's and at 8.5 g's most passengers would lose consciousness. The speed of the coaster can't be slowed down since then the coaster would follow a less steep projectile path and fall off the steeper tracks. The solution used in modern coasters is to create a Klothoid loop – a loop with a radius that starts wide at the bottom and ends narrow at the top.



The smaller the radius for a given speed, the greater the centripetal force ($f= ma = mv^2/r$). The wide radius at the bottom ensures the higher velocity needed at the bottom to get to the top will not create an excessively high centripetal force. The smaller radius at the top allows the coaster to travel at a slower speed but yet still have adequate centripetal force to complete the loop. Before the great breakthrough of Klothoid loops, earlier coasters using circular loops would often cause serious blackouts, or worse: snap passengers' necks. Still today it is too dangerous and difficult to build loops on wooden coasters.

4.5 Turns (and banked turns)

For a coaster to make a horizontal turn, the track must provide a sideways normal force N' as the necessary centripetal force. At the same time, the seat pushes up vertically with a normal force N equal to the weight of the rider. The free body diagram is shown below from the side view:



Vertically, the weight must equal the normal force N since there is no vertical acceleration. The seat pushes up on the rider and the rider pushes down with a force equal to the weight. Hence the vertical g-force is just one:

Vertically: Fnet = Wt – N = 0

N= Wt

So g-force = N / Wt

= Wt / Wt

= 1

Horizontally, the seat pushes inwards with a Normal force N' and the rider pushes back outwards with an equal force. Since this normal force N' is the only force responsible for making the coaster turn, it is also equal to the centripetal force.

Horizontally: Fnet = N'

ma = N'

So g-force = N / Wt

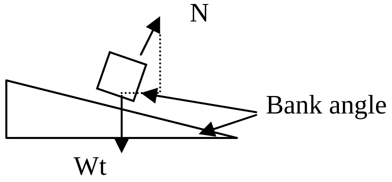
= ma / Wt

$$\boxed{\text{g-force} = a/g}$$

Recall the centripetal acceleration $a = v^2/r$. Thus the horizontal g-force will depend on how fast the coaster is moving and the radius of the turn. In a typical turn, the coaster may be moving at 30 m/s along a radius of 45 m. This corresponds to an acceleration a= 20 and a g-factor of 2.0. The same thrill could be found in a car, but not nearly as safely. If the friction between the tires and road needed to produce the centripetal force is inadequate, the car will take off in a straight line due to its inertia. It's a crash waiting to happen!

Banked turns: (the ultimate in comfort!)

A major safety improvement is to use banked turns. Turning on a hill means the normal force N is now pointing partly towards the center of the center and thus aids in the centripetal force. The steeper the bank, the more of the centripetal force that can come from the normal force N. In fact at the proper angle, it is possible to make a roller coaster than turns without the help of friction or a normal force N'. At this ideal angle, the free body diagram looks like:



The normal force has two functions. The vertical component offsets the weight while the horizontal component provides the centripetal force. Using geometry it is possible to show the angle in the smaller dotted triangle is the same as the bank angle. Then by trigonometry one can show

Nsin(angle) = Wt
Ncos(angle) = Fnet

Then N = Wt / sin(angle) or N = Fnet / cos (angle)

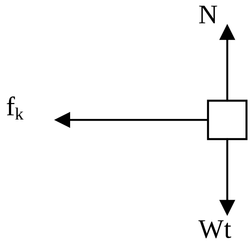
So g-force = N / Wt
= Wt /sin(angle) / Wt

$$\boxed{\text{g-force} = 1 / \sin(\text{angle})}$$

Unlike a flat horizontal turn, the g-force on a banked turn does not depend on the acceleration of the coaster. The g-force depends only on the angle of the bank. The greater the angle, the steeper the bank and the smaller the g-force. This is the ideal banked turn for which no additional forces are needed to steer the coaster – it drives itself around the corner!. Banked turns have also the advantage of lowering the g-force to more comfortable levels. Typical turns banked at 45 to 65 degrees produce g-forces between 1.4 and 1.1

4.6 Braking

Stopping a coaster is similar to starting one in that the vertical g-force is one and the horizontal g-force depends on the amount of acceleration. Stopping is different than starting in that the acceleration is negative (speed is slowed down) and the responsible force is the force of moving friction between the coaster and the brake pads. Hence the free body diagram resembles the following picture:



Newton's second law gives:
Vertically: $F_{net} = N – Wt = 0$
Horizontally $F_{net} = f_k = ma$

Vertically, the normal force is just equal to your own weight so the g-force is one:

g-force: Wt / Wt = 1 since N = Wt

Horizontally the seat is being pulled backwards away from you by friction while your own inertia carries you forward. You will feel a negative g-force of being pushed out of your seat when actually the seat is being pulled away from you. The force felt by the rider is just equal to the net force, which is the force of moving friction. Hence, the g-force will depend on the net deceleration.

Horizontally: g-force $= F_{net} / Wt$
$= ma / mg$
$= a / g$

Note that the value of acceleration (a) will be a negative number since the final speed is zero. Typically you feel a horizontal g-force of about – 1.5 when a coasters come to an abrupt stop. Thus you feel that you need a restraining force of about 1 and ½ times your weight to hold yourself in your seat. Seat belts must be used to provide a restraining force to prevent passengers from literally flying out of their seats! As previously mentioned, negative g's cause red out so coasters can't be designed to stop too quickly even if made possible using strong brakes and seatbelts.

5.0 Review Questions

1. How heavy do you feel at 3 g's? How heavy do you feel at 0.5 g's?
2. How are "red outs" different from "black outs"?
3. How are roller coasters designed to minimize the health hazards of large g forces?
4. What do you expect is the typical range of g's on a roller coaster?
5. What is the name of the force that prevents you from falling through the floor?
6. How does the direction of friction or air resistance compare to the direction of your motion?
7. Does inertia increase with more mass, more speed, or both?
8. Explain using inertia why it is more difficult for an elephant to make quick turns than a cheetah
9. Which coaster has more inertia, an empty coaster at rest or the same coaster at rest but loaded with passengers?
10. What are the two possibilities if the net force is zero?
11. If an object is accelerated in an upwards direction, what is the direction of the net force?
12. What is the acceleration of a 5000 kg coaster pushed by a net force of 25,000 N?
13. A 2000 kg coaster is accelerated from 0 to 20 m/s in 4 seconds. What was the net force used?
14. Centripetal forces cause objects to move in a circle. For each event, identify the centripetal force:
    a. roller coaster loop
    b. moon going around the Earth
    c. toy swinging on a string in a circle

15. What is the centripetal acceleration of a coaster which is moving at 40 m/s in a circular loop that has a diameter of 20 m?

16. Which direction would the earth follow if suddenly the sun was removed ?

17. A student swings a toy on a string over his head in a horizontal circle. What is the direction of the tension in the string?

18. A gun shoots off a bullet. Since the gun pushes out the bullet with the same force that the bullet pushes back on the gun, why don't the gun and bullet have the same acceleration?

19. The restraining bar in the ride Shockwave hits your head. What is the reaction force?

20. Use action-reaction pairs to explain what happens if you were in outer space and pushed off from the side of your space vessel?

21. As the Space Shuttle orbits our planet, it is actually free falling around the earth. Why does this not mean the Shuttle has no weight?

22. Imagine you are standing on a weigh scale while free falling inside an elevator. If your mass is 90 kg, what would the scale read? What is your true weight?

23. How is it possible to be truly weightless (not just feel weightless)?

24. What is the one force that must be missing in order to feel weightless, for example on Giant Drop?

25. What is the acceleration of a 1000 kg elevator supported by an 8800 Newton(N) normal force?

26. How is the direction of the normal force determined?

27. What happens to you on a trampoline if your normal force is greater than your weight?

28. Where do you expect on a roller coaster the normal force is greater than the weight?

29. When is the only time on a ride where static not moving friction must be overcome?

30. If a coaster is moving up a hill, what direction is the force of friction?

31. Modern coasters use rubber instead of steel wheels to roll against the steel track. Which type of wheel would you expect to have a lower coefficient of friction and why?

32. Which friction, static or kinetic, holds a parked car from sliding down a steep hill?

33. What would happen to the force of friction if the normal force is doubled?

34. Is it the centripetal or centrifugal force that makes circular motion possible?

35. Why does a coaster have a tendency to go straight when making a turn?

36. If g-force is not actually a force, then what does it measure?

37. At positive g's, do you feel heavier or lighter?

38. At what g force can you feel absolutely nothing?

39. Do you get bumped out of your seat or pushed into when experiencing negative g’s?

40. What happens to the g-force you feel if the normal force on you were to double?

41. Where on a coaster ride will you most likely experience positve g’s? negative g’s?zero g’s?

42. What is the acceleration of a coaster at the start if it begins moving with a g-force of 0.8?

43. What is the normal force of a 5100 kg coaster while at rest on a flat section of track?

44. What g-force would you feel on a new coaster which is accelerated from 0 to 40 m/s in 2 s?

45. A hill is designed to produce a feeling of weightlessness. How should the shape of the hill be changed if later it’s found out the coaster speed is slower than expected?

46. If a Giant Drop rider is truly free falling from rest, what speed should she have after 2 seconds?

47. What danger exists if a coaster hill is not curved enough?

48. In which seat will you be traveling fastest when riding a coaster over a hill?

49. What g-force do you feel if riding at 25 m/s over a hill of radius 40 m?

50. What can happen to a coaster car when experiencing negative g’s?

51. Why is no forward force needed to keep a coaster moving when traveling into a loop?

52. How does the speed of an actual coaster change as it completes a loop?

53. How do the g-forces change as a coaster goes from the bottom to the top of the loop?

54. If at the top of the loop the normal force is zero, what force must provide the centripetal acceleration to complete the turn?

55. For a circular loop of constant radius, if the acceleration is 9.8, what is the g-force at the bottom, side,and top of the loop? Would you ride this loop?

56. If a piece of track suddenly broke along a loop, which direction would the coaster move towards?

57. It is possible in a coaster to fall faster than the acceleration due to gravity. How is this possible?

58. How does the radius at the bottom of a Klothoid loop compare to that at the top?

59. Why are Klothoid loops preferred over circular loops?

60. What are the advantages of banked turns over flat horizontal turns