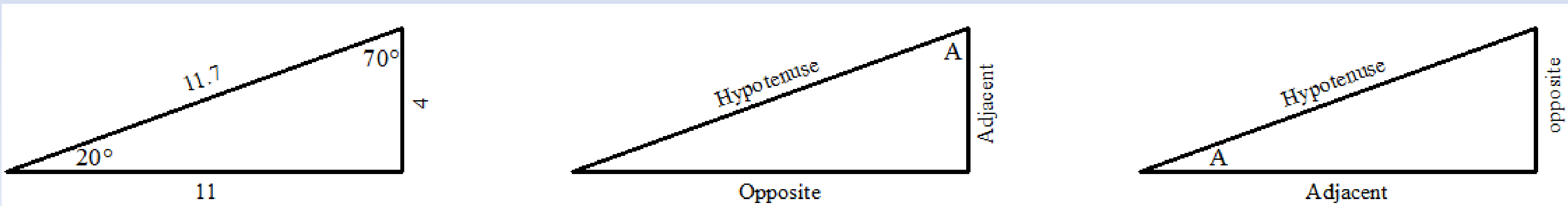
Themed: 11-01

Projectile Motion

Trig, Direction methods & scaling, 2-d
Nifty, solving horizontal projectiles

Trig Review

- TRIg is about right TRIangles
- A single side and an angle, or two sides define all sides and all angles
- Trig functions: put an angle in a ratio out
- Inverse trig functions: ratio in, get angle out
- Common error: opposite and adjacent mix-up (middle example)



What is a projectile?
How is this unit different?

- Motion in a plane (not in a straight line)
- A projectile is "a moving object whose motion is only affected by gravity"
  - Sound more like freefall or terminal velocity to you?
- Very different because two dimensions instead of one (not linear any more!)
- First: need rules on describing directions:

Describing Vector Direction

- Two ways of specifying vector direction
  1. Relative to horizon
  2. Mapping method
- You need to be able to recognize both
- You should be able to use both, but if you must be good at one, Mapping Method is more useful – know it

Vector Descriptions - Horizon Method

- Assume: Archer at right shoots her arrow at 75 m/s
- Horizon method says:

Speed + degrees above/below horizon

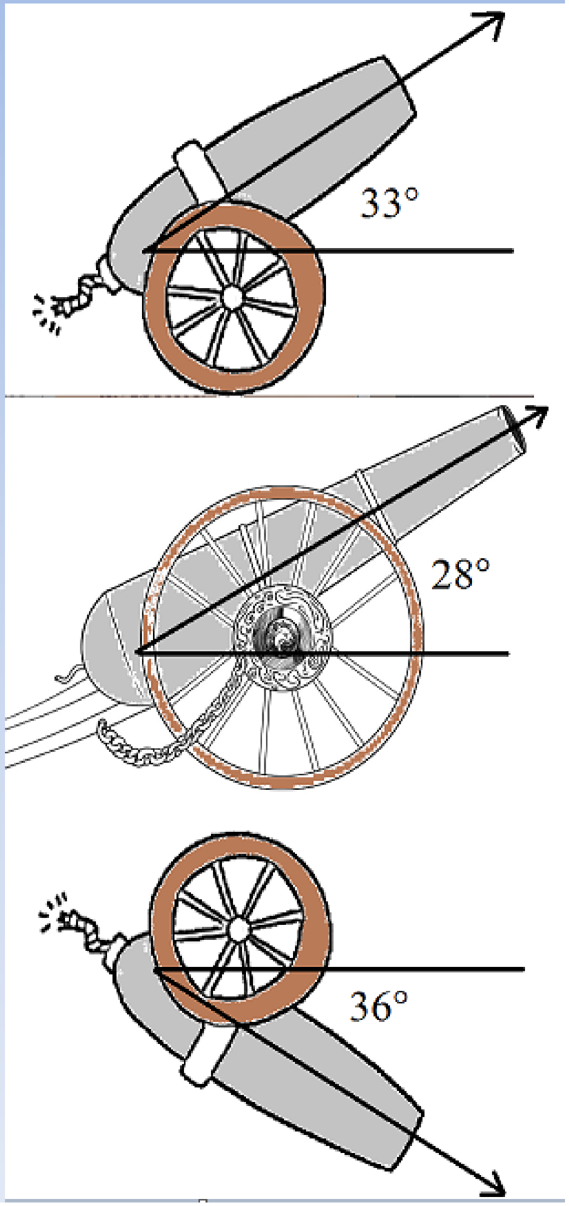
In this case:

V = 75 m/s 37° above horizon

Archer above is the women's world record holder for distance, April Moon

Vector Descriptions - Horizon Method

- Vectors are drawn with arrows, or northeast would be identical to southwest!
- Apply this method to find initial velocities at right
- Why do we call it, “initial”? Why would it change?

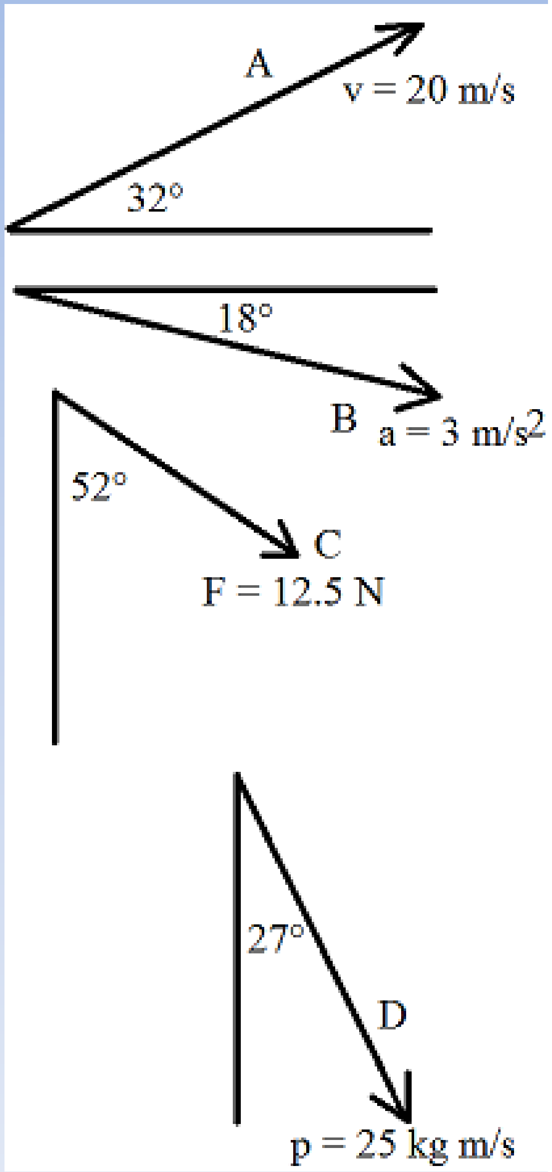


Vector Descriptions – Mapping Method

- Mapping method says:

Speed + degrees direction 1 of direction 2

- Directions are N, E, S, W
- Example: 30° N of E describes: East-pointing arrow rotated by 30° toward North
- Complete vector might be: 3.5 m/s 30° N of E
- Try the examples at right including “2nd method” (complementary angle)



Scales/Scaling

- Product of a vector and scalar is a vector
  - Example: doubling your velocity is multiplying velocity vector by scalar (2), result is a vector!
- Used to represent something very big or very small
  - Maps
  - Amoebas
- Pick a scale by ratio of actual size to room you have to represent it
  - Example: You have 10 cm width on paper and USA is 3000 miles wide: Your scale is 300 miles = 1 cm

Nifty (TNEOM)…2-d Version

- Notice we now have y AND x?
- Y will mean vertical from now on and X horizontal
- Good news: We have no new equations! None!
- Bad news: YOUR equations still say X, you will need to know when they mean Y
- Good news: We only have ONE equation for X (so all the rest must mean Y!)

Y-Equation	a	t	v_i	v_f	Δy
$\Delta y = \frac{(v_i + v_f)}{2} t$	☹	√	√	√	√
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√
$\Delta y = v_i t + ½at^2$	√	√	√	☹	√
$vf = vi + at$	√	√	√	√	☹

X direction (where velocity is <u>not</u> changing): $\Delta x = v_x t$

Nifty (TNEOM)…2-d Version

- Time (t) is the common link between X and Y directions
- There is a vi (v_{yi}) and vf (v_{yf}) in Y, but not in X…can anyone clever figure out why?
- Patterns are exactly the same: Each equation only involves 4 of the 5 variables
- Huge concept: X and Y are independent!
  - you can examine one independently of the other
- Bad news: Table (below) not yours on test; look at packet eqn. list & find equivalent eqns. for all 4 below

Y-Equation	a	t	v_i	v_f	Δy
$\Delta y = \frac{(v_i + v_f)}{2} t$	☹	√	√	√	√
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√
$\Delta y = v_i t + \frac{1}{2}at^2$	√	√	√	☹	√
$vf = vi + at$	√	√	√	√	☹

X direction (where velocity is <u>not</u> changing): $\Delta x = v_x t$

Horizontal Projectiles

- Vyi = zero….’nuff said?
- Combine this concept with independence of X and Y to solve this:

If a bullet is dropped from 2 m high and a bullet is fired at 300 m/s from 2 m high, which will strike the ground first?

- What’s happening in the Y direction in both cases (list all five variables and solve for time)
  - What does the physics tell you will happen?

Themed: 11-02

Definitions, NIFTY tips, motion changes & angle variation, resolving vectors and solving angled projectiles

Vectors & Projectiles definitions

- **Projectile Motion** – Two-dimensional motion only affected by gravity (zero air resistance is assumed)
- **Resolve** – Process of finding the X and Y components of a vector
- **Resultant** – The sum of two or more vectors
- **Horizon** – Line where the earth appears to meet the sky
- **Ground speed/velocity** – motion of an object relative to the ground
- **Air speed/velocity** – motion of an object relative to the air it's moving in
- **Wind speed/velocity** – motion of air relative to the ground
- **Range** – The horizontal distance a projectile travels (Δx)

Solving Projectile Motion Problems

- X and Y motion are at right angles (independent of each other)
- Time (t) is the variable they have in common
- Nifty works the same as before (four variable, you must know 3, sad face our same friend)
- One X direction equation (must know two to use)
- Vx doesn't change, Vy does (that why there's Vyi and Vyf)
- Symmetry assumptions (same time up as down, final speed = initial speed, same distance up as distance down)

Y direction (where velocity is changing)

Equation	a	*t*	v_i	v_f	Δy
$v_f = v_i + at$	√	√	√	√	☹
$\Delta y = \frac{(v_i + v_f)}{2} t$	☹	√	√	√	√
$\Delta y = v_i t + \frac{1}{2}at^2$	√	√	√	☹	√
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√

X direction (where velocity is not changing): $\Delta x = v_x t$

Solving Projectile Motion Problems

- Two kinds of problems:
  - Horizontal motion problems
    - Vyi = 0
    - Shoots a gun horizontally, rolls off of a table, etc.
  - Projectiles shot at angles….hard!
- Two part problems:
  - Use X equation to find t, then use t as third Y direction known
  - Use Nifty to find t, then use t as second X direction known

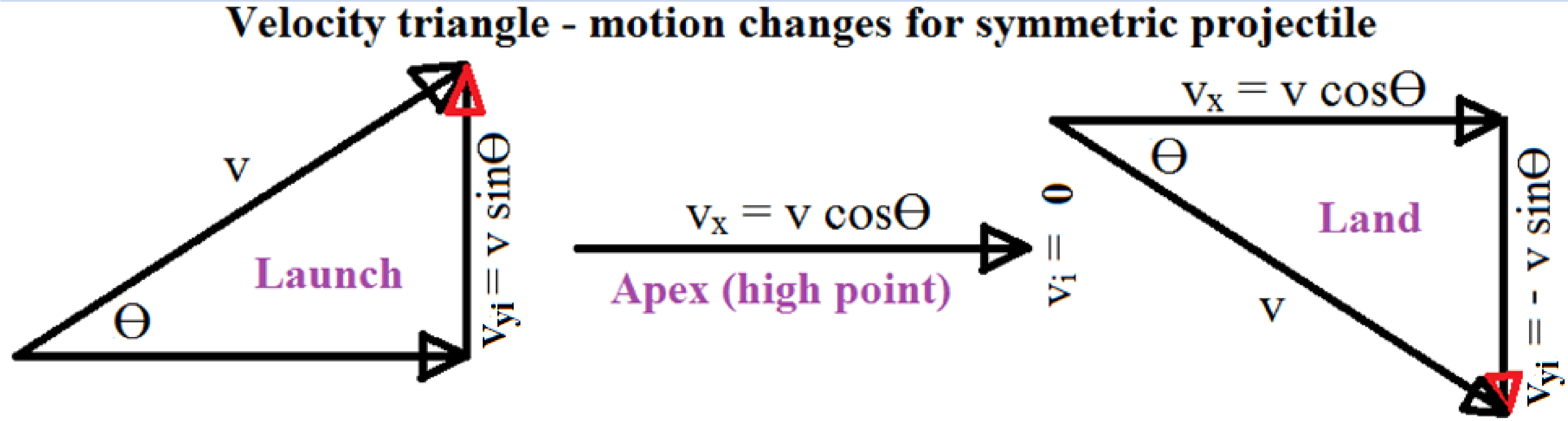
Y direction (where velocity is <u>changing</u>)

Equation	a	t	v_i	v_f	Δy
$v_f = v_i + at$	√	√	√	√	☹
$\Delta y = \frac{(v_i + v_f)}{2} t$	☹	√	√	√	√
$\Delta y = v_i t + \frac{1}{2} a t^2$	√	√	√	☹	√
$v_f^2 = v_i^2 + 2a\Delta y$	√	☹	√	√	√

X direction (where velocity is <u>not</u> changing): $\Delta x = v_x t$

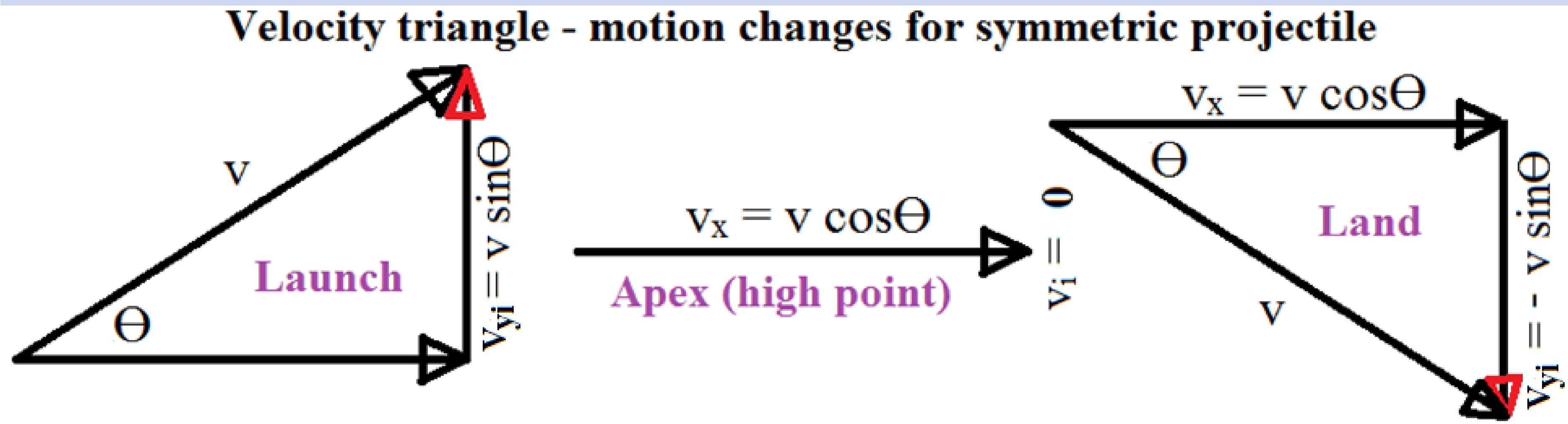
How motion changes during projectile's lifetime

- When is speed a maximum? A minimum?
- Is speed increasing with altitude gain? Why?
- If v>0, what angles produce zero range?
- What angle produces min and max hang time?
- What angle is "best" compromise between big Vx and big hang time producing biggest range?



More conclusions: Effects of angle variations

- Small angles = big Vx and make rapid X directions progress, but don't stay airborne long, so small range
- Big angles = big hang times, stay up a long time, but have tiny Vx, so small range
- Best range is equal compromise between hang time and big Vx (45° angle)
- Complementary angles have equal trade-offs, so have <u>equal</u> ranges
- See how Vy=0 at max height? Does the acceleration switch directions when that happens?



Effect of angle above horizon on…

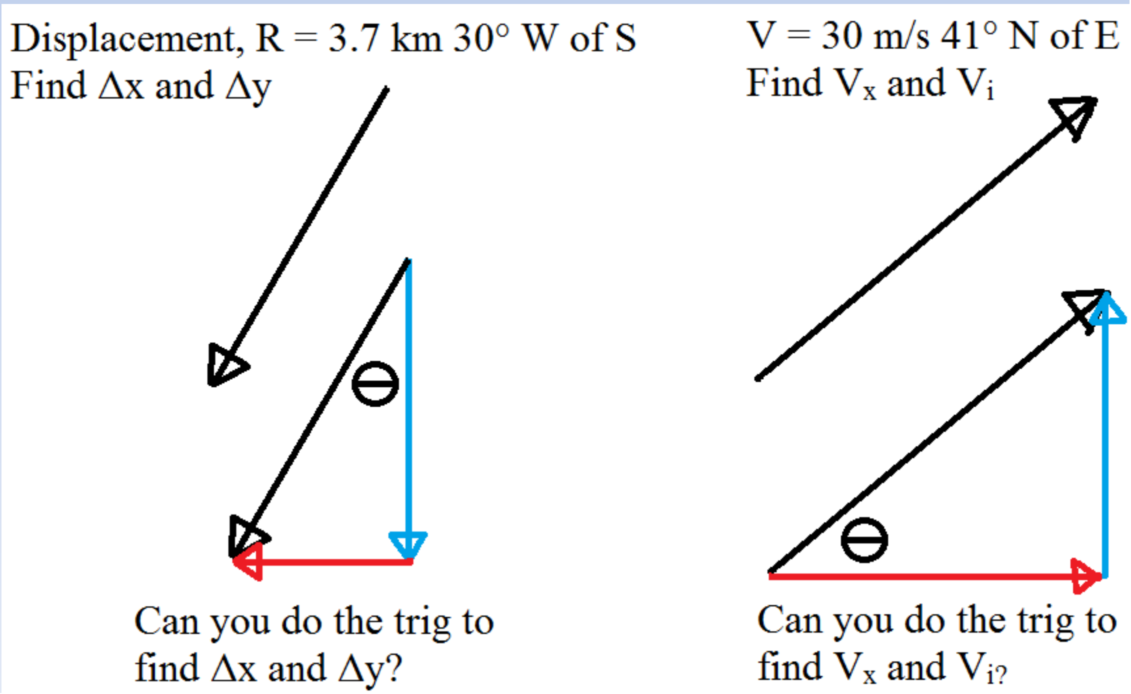
- What angle(s)
  - Maximizes hang time?
  - Maximizes range?
  - Give equal ranges?
  - Maximizes peak height?
  - Maximizes Vx?
  - Makes a football go farthest?
  - Tennis players only: Is hardest for a tennis player to hit? (Think John Isner vs. Andy Roddick)

Resolving Vectors

- Finding the x and y components of a vector
- Useful to find Vyi and Vx (why do you want Vyi and Vx anyway?)
- For finding resultants (adding vectors)

Resolving Vectors

- Process:
  1. Sketch vector to be resolved
  2. Vector becomes hypotenuse, finish triangle (one side is totally x and the other side is totally y)
  3. Use trig to find values (careful!)

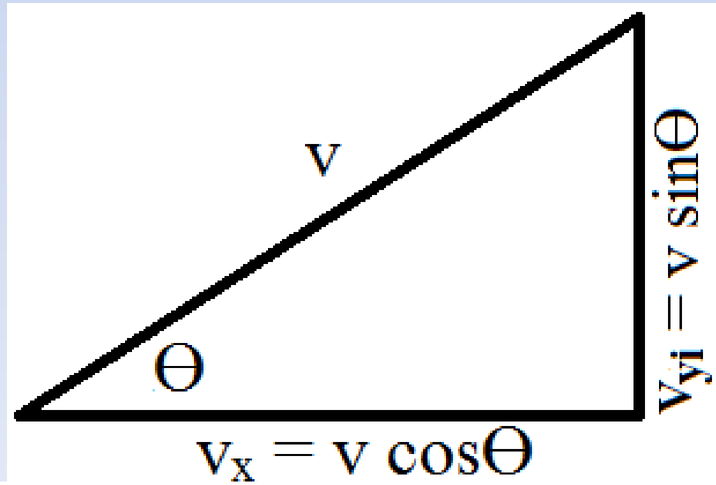


Solving Projectiles At Angles

- What the heck does this even mean?
- Examples:
  – Tom throws a ball 30 m/s at an angle of 20° above horizontal from an initial height of 1.4 m. How far will the ball go when striking the ground?
  – What minimum speed must a pass be thrown to reach a receiver 30 m downfield when caught at the same height as thrown? (YES, this implies a specific angle!)
- We're not solving these *yet*!

Solving Projectiles At Angles

- Find as many of X/Y motion variables as possible
  - If you have two horizontal knowns, you can find 3rd
  - If you have three vertical knowns, you can find other 2
- If possible, RESOLVE the velocity vector in the x-direction and y-direction components
- Takes place on earth?
- ΔY is negative if down, 0 if same level
- Time ties X and Y together
- What equation can I use?
- HARD: Algebra substitution



Solving Projectiles At Angles

- Wanna try these now? (*harder than horizontal*)
  - The strongest arms out there can hurl a baseball about 45 m/s (100 mph). Assuming projectile motion, what distance could this baseball go?
  - If a soccer ball is kicked at a 30° angle at 30 m/s and is considered to be a projectile, what is the hang time of the ball and its final velocity? *(answers to these are on the next slide)*

Solving Projectiles At Angles

- Wanna try these now? (*harder than horizontal*)
  - The strongest arms out there can hurl a baseball about 45 m/s (100 mph). Assuming projectile motion, what distance could this baseball go? *(206 m)*
  - If a soccer ball is kicked at a 30° angle at 30 m/s and is considered to be a projectile, what is the hang time of the ball and its final velocity? *(3.06 sec, Vf = 30 m/s, 30° below the horizon)*

Solving Projectiles At Angles Challenging problems!

- Wanna try these now? (*harder than horizontal*)
  - Tom throws a ball 30 m/s at an angle of 20° above horizontal from an initial height of 1.4 m. How far will the ball go when striking the ground?...*More than one way to do it, use $t_{up} + t_{dn}$ or use quadratic (62.7 m, t = 2.22 sec (t_{up} = 1.05 sec, t_{dn} = 1.17 sec))*
  - What minimum speed must a pass be thrown to reach a receiver 30 m downfield when caught at the same height as thrown? (YES, this implies a specific angle!)…*hint: solve for t in terms of V using X-stuff*
  - *($V_{minimum}$ = 17.15 m/s)*

Themed: 11-03

Projectile Motion

Adding Vectors Mathematically to find Resultants ("Hardest thing in the world")

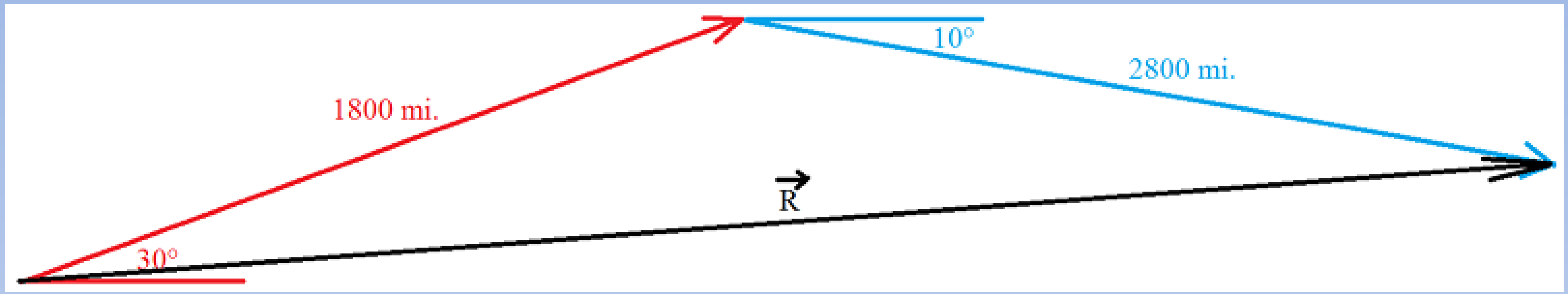
Finding Resultants Mathematically

- Make a sketch
- Resolve each vector for X and Y
- IMPORTANT: Keep + or – correct
- Sum all X values in X column & Y values in Y column
- Find, R, the resultant vector using
  - Pythagorus
  - Trig

Finding Resultants Mathematically

- Example: Alex takes a trip to France! On the first part of his trip, his displacement is 1800 miles, 30° N of E. On the second part of his trip, his displacement is 2800 miles, 10° S of E. Find the resultant, R, that describes his overall displacement.

Find Resultant: Draw a picture, make a table



- Graphically:
  - Add head-to-tail
  - R: from tail of 1st to head of 2nd
  - Order doesn't matter (parallelogram rule)
- Mathematically:
  - Call 1st vector P and 2nd Q
  - Resolve P and Q
  - Find total X and Y
  - Pythag finds magnitude of R
  - Trig finds angle of R

$P_x = 1800 \cos 30° = +1559$ mi
$P_y = 1800 \sin 30° = +900$ mi
$Q_x = 2800 \cos 10° = +2757$ mi
$Q_y = 2800 \sin 10° = -486$ mi

	X	Y
P	+1559	+900
Q	+2757	-486
Tot	**+4316 mi**	+414 **mi**

Pythagorus: $R^2 = X^2 + Y^2 = \mathbf{4316}^2 + 414^2$, $R = $ **4336 mi**

$\Theta = \text{Tan}^{-1}(414/4316) = 5.5°$

R = **4336 mi** , **5.5°** N of E

Try another?

- These problems are so much fun, they're like Saturday at your BFF's house!
- Relative to the wind it's in, a plane is flying at 120 m/s, 40° S of W. The wind velocity is 50 m/s, 10°E of S. What is the ground velocity of this plane?

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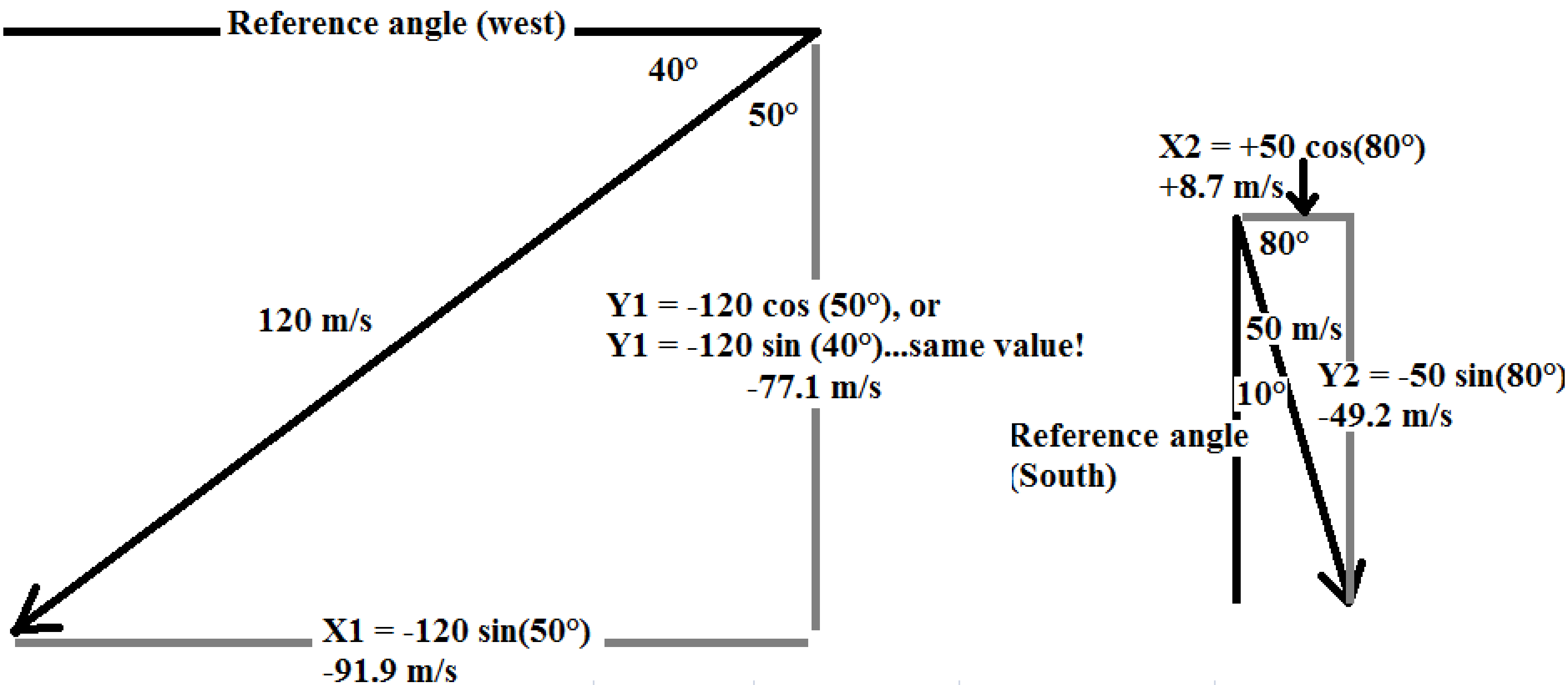
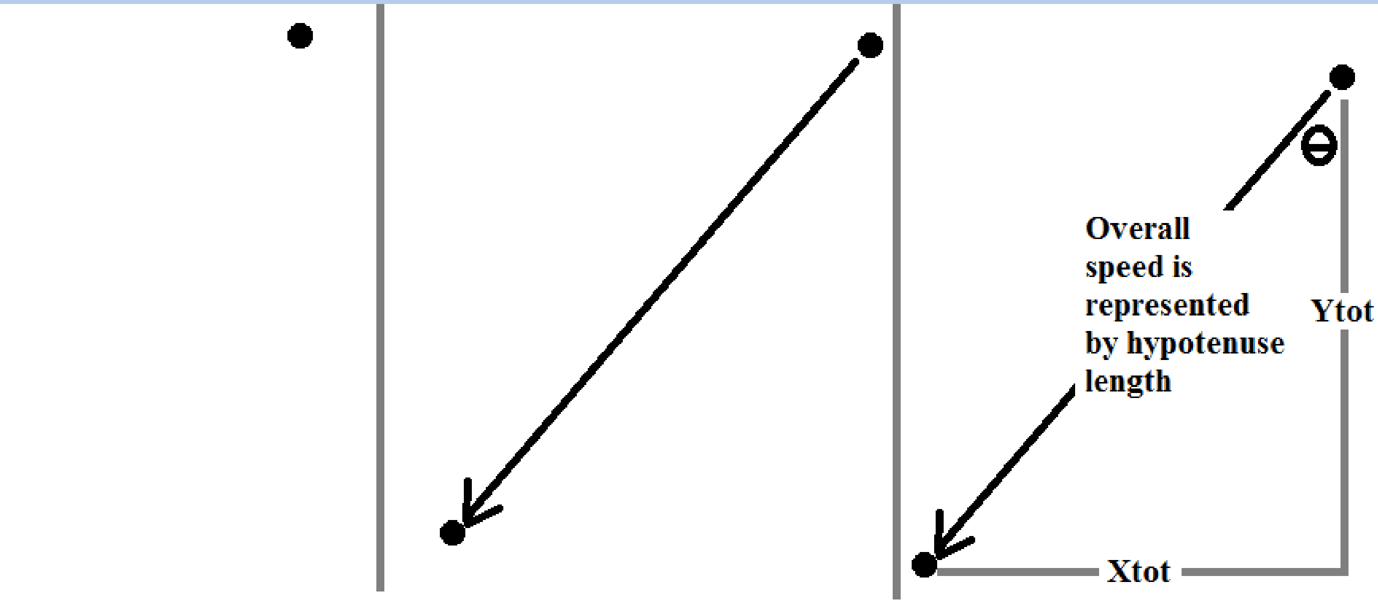


	X	Y
1	-91.9	-77.1
2	+8.7	-49.2
Tot	-83.2	-126.3

Now, how do I make a vector from my total results?

- First, look at your totals. Your X total is negative, so your resultant (total) will have to go left. Leave room for going left.
- Second, Your Y total is negative, so your resultant will have to go downward. Leave room for going downward.
- To have room to go left and downward, pick a starting point up and right…



Speed: $R = \sqrt{(83.2^2 + 126.3^2)} = 151.2$ m/s

Direction: $\Theta = \tan^{-1}(83.2/126.3) = 33.4°$

Resultant = 151.2 m/s 33.4° W of S

Uno mas?

- Want to: Amaze your friends? Strike fear in your enemies? Get the attention of that person you have a crush on? Adding vectors to find resultants will do all that and more!
- A river flows at 4.5 m/s, 60° N of W. A boat moves relative to the river at 6.0 m/s, 30°S of E. What's the velocity of boat relative to the shore?

	X	Y
1	-2.25	+3.90
2	+5.20	+3.00
Tot	+2.95	+6.90

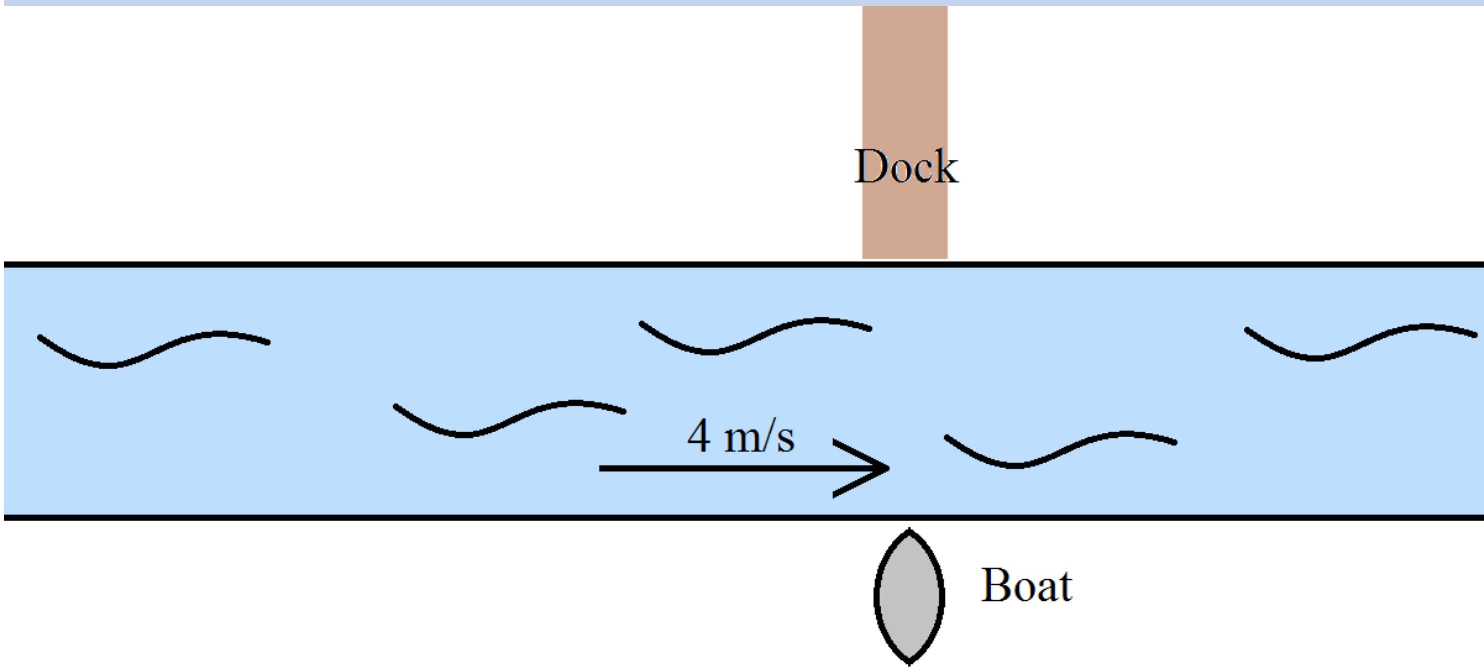
Speed: 7.50 m/s

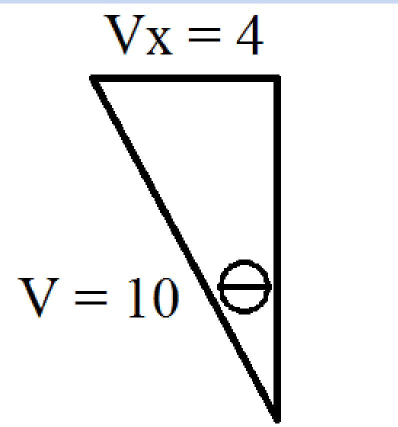
Direction: Θ = 66.9° S of E…SAME as Θ = 23.1° E of S

Resultant = 7.50 m/s 66.9° S of E

Crossing a river in a boat

- A boat needs to directly cross a river (below). The boat will move 10 m/s relative to the water stream. What launch angle is needed?
- Solution: In order to cross DIRECTLY, the boat must be tilted to the left so that it will have a westward velocity component of 4 m/s. This will exactly cancel out the eastward 4 m/s stream velocity, resulting in a direct crossing! The overall speed is the hypotenuse (10 m/s)



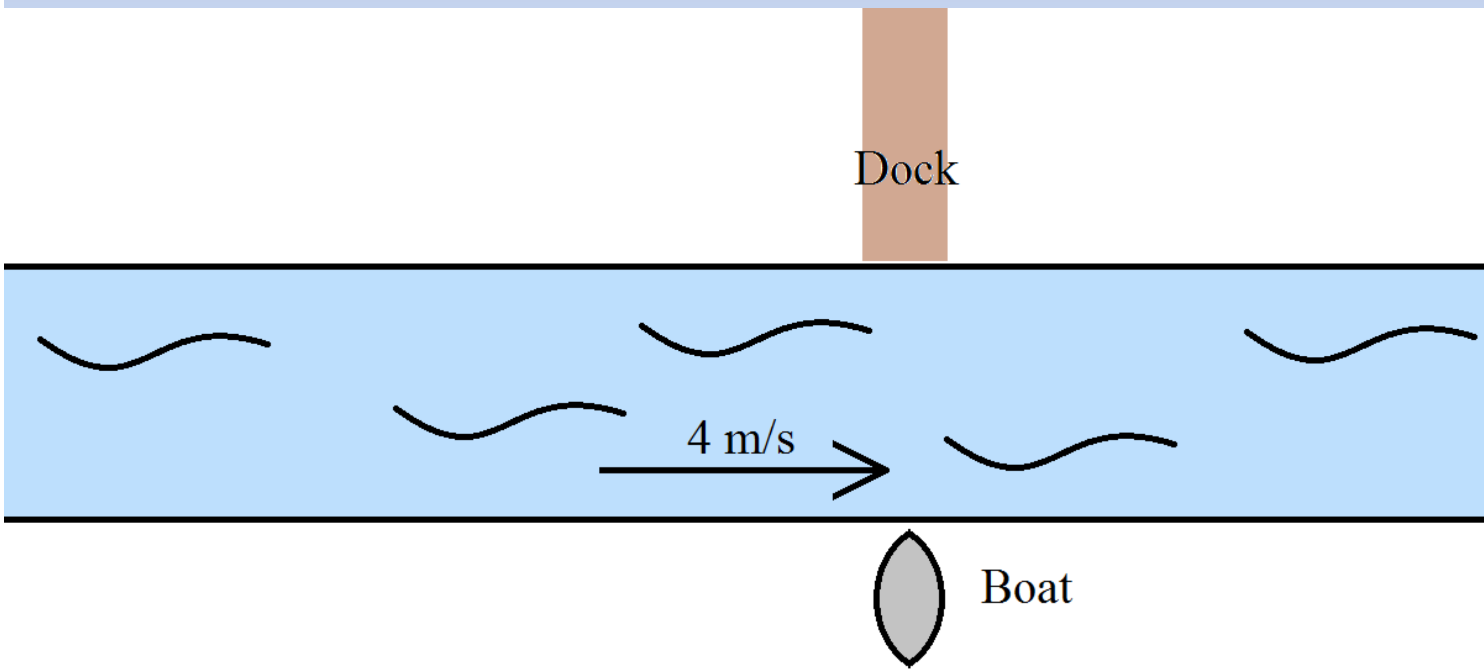


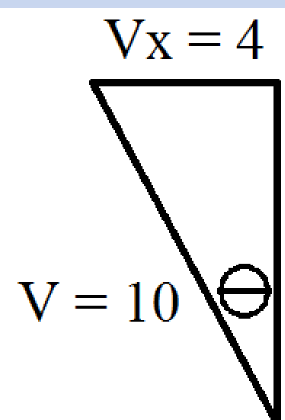
θ = Sin (4/10)

θ = 23.6° W of N

Crossing the river…how long to cross?

- How long will it take to cross the river to reach the dock? (assume the river is 200 m wide)
- ΔY = 200, Vy = 10 m/s Cos (23.6) = 9.16 m/s
- Vy represents the rate of northward progress
- Your displacement will be 200 m north
- ΔY = Vy x Δt → Δt = 200/9.16 = **<u>21.8 s</u>**





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