# 5. Baseball Drag and Golf Drives

## Vectors and Scalars

### 5.1 Vectors and Scalars

Physical quantities differ according to their complexity. For example, the ratio of the circumference of a circle to its diameter is π, a pure number with no units or dimensions. Yet it refers to a common physical curve, the circle. We develop categories for physical quantities to help us organize our thinking, and to improve our description of mature. Pure numbers, like π, contitiute the first category.

**Pure Numbers** – numbers such as π, 4, etc, have no units or dimensions.

Another group of physical quantities is closely related to the group of pure numbers. Members of this group require a unit in addition to the number for their specification. Together, the number and unit are called the "magnitude" of the quantity.

**Magnitude** – the size of the number plus a unit. An example is a length like 12 m.

Quantities which are specified by a number alone, or by a number and unit, are called "scalars." Scalars have a magnitude and require only one number for their specification.

**Scalar** – a quantity specified by its magnitude; it can be a pure number or have units. Examples are π or a mass like 7 kg.

Some physical quantities we encounter are too complex to be specified completely by a number and a unit. For example, 6 mi southwest and 6 mi west have the same magnitude (6 mi) but clearly differ. Travel 6 mi west when you are supposed to go 6 mi southwest and you will end up in the wrong place. Displacement is an example of a physical quantity that is incorrectly specified by a magnitude alone (6 mi) and must have a direction (southwest) to complete the quantity. Quantities that require both magnitude and direction for their specification are called "vectors."

**Vector** – any quantity that has direction in addition to magnitude.

We will encounter many scalar and vector quantities during our study of Physics. Examples are given below:

| Sample Vectors | Sample Scalars |
|---|---|
| Displacement | Length |
| Velocity | Mass |
| Acceleration | Time |
| Force | Speed |
| Liner Momentum | Work |
| Angular Momentum | Power |
| Torque | Kinetic Energy |
| | Potential Energy |
| | Moment of Inertia |

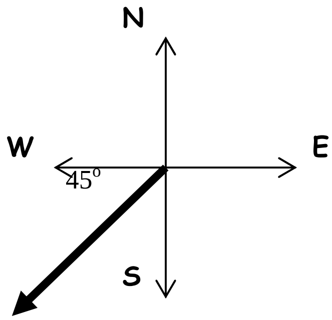
According to the definition the magnitude of a vector is a scalar. Sometimes these magnitudes are interesting in their own right. Quite often, for instance, we are interested in the magnitude of the displacement (length).

We use vectors because they simplify. Once we learn to manipulate vectors all vector physical quantities can be handled in the same way. This illustrates the point made earlier: sorting quantities into categories helps to organize our thinking.

Before we can manipulate vectors we must represent them. We use a simple geometrical method which represents a vector by an arrow. The length of the arrow represents the magnitude and the vector's direction is represented by the directional orientation of the arrow.

**Example – Displacement Vector**

To represent the displacement from Hinsdale Central to Argonne National Laboratory, we draw an arrow. The direction of the arrow is (roughly) 45 degrees southwest, and its construction requires the use of a protractor. The length of the arrow is 6 miles, and its construction requires a ruler and a scale calibration (such as 1 cm = 2 mile).

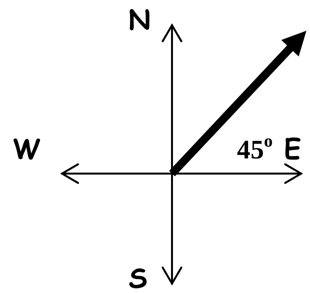



It looks something like this, only drawn to scale. Vectors don't care what road you have to drive to get there – vectors are more like the path you could fly in a helicopter to get to the end point.

Displacement is measured in length units, so representing a displacement vector is similar to using a map. Every map has a legend which represents the measured distances on the map (in inches) to the actual distance along the road (in miles). The scale calibration performs the same function for the vector displacement. If, instead of a length, the vector is a velocity (or force or whatever) the same function must be preserved. A scale calibration must be provided which converts velocity (or force or whatever) units to length units.

**Example – Velocity Vector**

To represent the velocity of a runner sprinting at 12 mi/hr to the northeast, we draw the arrow shown below. Again, a protractor and ruler are used. In this case, the scale calibration relates the ruler measurement (1 in) to the velocity (12 mi/hr).

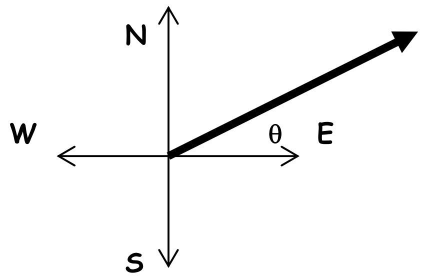



(My diagrams are slightly wrong because they are drawn on the computer. Accurate measures of length and angle must be made.)

These examples show how to draw a vector when the magnitude and direction of a physical quantity are known. The reverse procedure is just as simple. To determine the magnitude and direction of a vector from measurements on the arrow we just reverse the process. To obtain the direction we measure the angle with a protractor. To obtain the magnitude two steps are needed. First we measure the length of the arrow in a convenient unit such as centimeters or meters. Then we use the scale calibration (such as 1 inch = 50 Newtons) to determine the magnitude.

**Example – Force Measurement**

Here, we have drawn a force vector, and we wish to determine its magnitude and direction. Using a protractor we measure θ = 30° north of east, and using a ruler we measure 2 inches for its length. For a scale calibration, (1 in = 50 N) we obtain F = 100 N.




(My diagrams are slightly wrong because they are drawn on the computer. Accurate measures of length and angle must be made.)

## 5.2 Multiplication by a Scalar

Now that we know how to represent vectors we are ready to learn how to manipulate them. Vectors are arrows, so we need rules which tell us how to manipulate arrows.

Some physical quantities are defined to be the product of a vector and a scalar. For example, the vector called "linear momentum" is defined as the product of mass times velocity – and we know that mass is a scalar while velocity is a vector. Our equation is:

$$\mathbf{p} = m\,\mathbf{v}$$

How are the magnitude and direction of **p** to be defined? We need a rule which determines the magnitude and direction of the new vector in terms of the scalar and the old vector.

Actually the rule is simple. The direction of the new vector is the same as the direction of the old vector, since the scalar had no direction at all anyway. Multiplication of a vector with a scalar does not change the direction. The magnitude of the new vector is the product of the scalar and the magnitude of the old vector.

Although multiplication of a vector by a scalar does not change the direction, changes in the units and dimensions do occur, and thus the scale calibration is changed.

**Example – the Linear Momentum Vector**

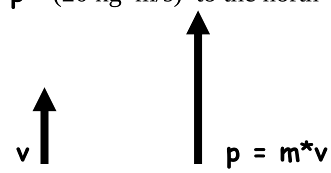
Suppose we have a velocity (**v** = 10 m/s to the north) and a mass **m** = 2 kg.

$$\mathbf{p} = m\,\mathbf{v}$$

**p** = (2 kg)*(10 m/s to the north)

**p** = (2 kg)*(10 m/s) to the north

**p** = (20 kg*m/s) to the north

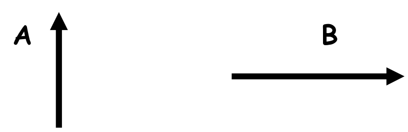



We note that the new vector is two times larger since the scalar had a size of 2. The vector could be drawn the same size if we changed the scale calibration to reflect this.

## 5.3 Vector Addition

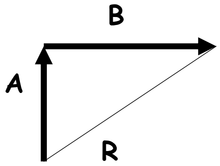
Two or more vectors may be added together only if the vectors are the same kind. They must both be velocities, or both be forces, or whatever, but addition of a force to a velocity makes no sense. Vectors may be moved parallel to themselves without changing their magnitude or direction. This geometric property is important to the geometrical addition of arrows.

Vectors are arrows. To define vector addition we define how arrows are to be added. Consider the two arrows (vectors) **A** and **B** shown below:



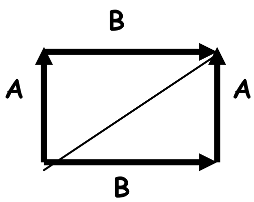

We wish to define the vector sum (or "resultant vector") **R** such that we give meaning to the equation:

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$




Where **R** begins at the start of ***A*** and ends at the end of **B**. We obtain the vector sum by adding the two or more vectors "tail to tip" which means that the tail of the second vector begins at the tip of the first. It is interesting to note that the order that we add vectors is not important.

***A*** **+ B = B +** ***A***



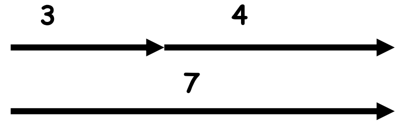

**Example – Vector Addition**

Suppose that the two vectors above represent displacements. ***A*** = 3 m north and **B** = 4 m east. If we draw them to scale, we can measure the resultant vector **R** and we get **R** = 5 m. (Don't forget to use a scale conversion.) We measure the angle with the protractor and get either 53$^o$ to the east of north or 37$^o$ to the north of east. Now we have both a magnitude and a direction for **R**, and this means that we have completely solved for the vector answer.

**R** = 5 m at 37$^o$ to the north of east.

Vector addition can be similar to, or quite different from, ordinary addition. To dramatize these similarities and differences we consider adding vectors which make special angles with each other.

When two vectors are parallel, vector addition corresponds to ordinary addition. See the two vectors below? The sum of **(3+4)** adds as you might expect:



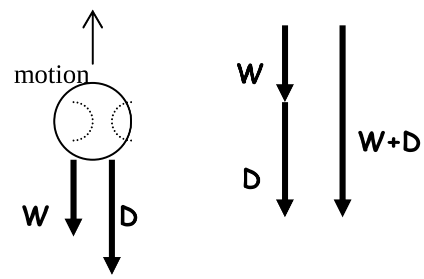

Vector **7** has the same magnitude as **(3+4)** as given by ordinary addition.

**Example – Drag on a Baseball**

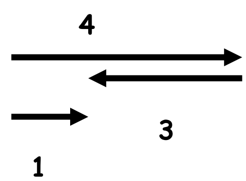
Two forces act on a baseball moving upward as shown below. Both the weight **W** and the drag **D** are downward. The vector sum of those two parallel forces

**F = W + D**

has the same magnitude as the ordinary sum.




When two vectors are oppositely directed (anti-parallel) addition corresponds to ordinary subtraction. Look at how the vector addition changes when we add **(4+ -3)**



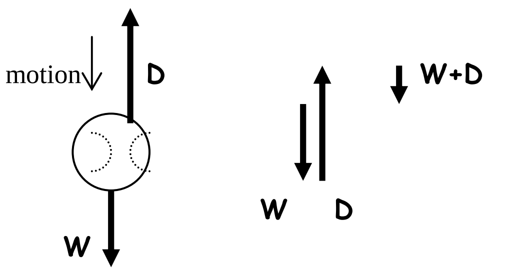

and we get an answer of one, just like we would have gotten with ordinary subtraction.

**Example – A Falling Baseball**

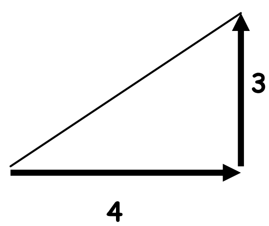
Two forces act on a falling baseball. The weight **W** is still down, but and the drag **D** is now up. The vector sum of those two parallel forces

**F = W + D**

has the same magnitude as **W - D**.




When two vectors are perpendicular, addition does not correspond to ordinary algebraic sum or algebraic difference. Below we see two vectors that add together as a triangle when connected tail to tip.




Using the Pythagorean Theorem we find the magnitude of the vector sum to be 5. Using a protractor we find the angle between the 5 and 4 vectors to be about 37o. These geometrical results correspond to the vector equation:

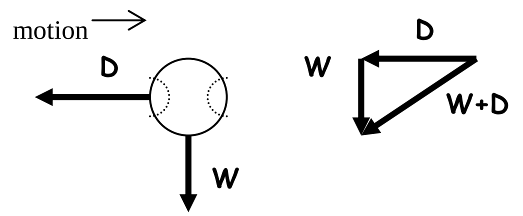
**3 + 4 = 5**

**Example – A Line Drive**

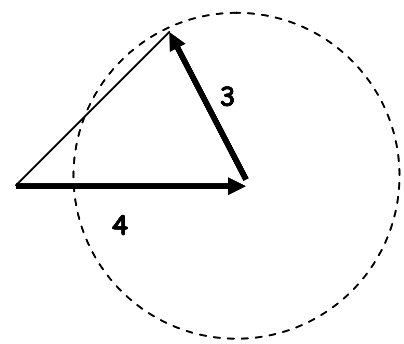
Consider a line drive, a horizontally batted baseball. Two forces act on the ball. The weight **W** is still down, but and the drag **D** is now in the horizontal direction against the motion. The vector sum of those two perpendicular forces

**F = W + D**

has the same magnitude as $\sqrt{(W^2+D^2)}$ from the Pythagorean theorem.



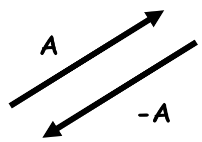

The sum of the vectors **3** and **4** in the general case will have a magnitude somewhere between their sum of **7** and their difference of **1**. The range of possibilities can be seen geometrically by first fixing one vector (the **4**) and considering all possible directions that the other (the **3**) can connect "tail to tip." The head of the **3** forms a circle shown below:




Thus, if **R = A+B**, then the magnitude of R will be somewhere between the magnitude **|A+B|** and the magnitude **|A-B|**.

## 5.4 Vector Subtraction

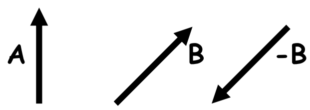
Before introducing vector subtraction, let's consider the meaning of a vector **(-A)** in terms of the vector **+A**. We define the negative of a vector to be a vector of the same magnitude but opposite direction, as shown below:

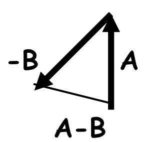



In vector subtraction we take the difference in two vectors, or the sum of one vector plus the negative vector of the second:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

This actually means that we can subtract vectors in exactly the same way that we added them earlier! This is done in two steps: (1) the negative of the second vector is formed, and (2) it is added to the first vector.

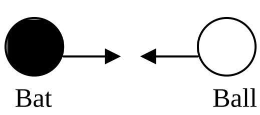



**Example – Relative Velocity**

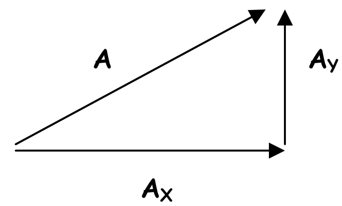
The velocity of a baseball relative to the bat is defined to be the vector difference between the baseball velocity $\mathbf{v}_{BALL}$ and the bat velocity $\mathbf{v}_{BAT}$. Consider a ball moving at 90 mi/hr moving toward a batter who achieves a bat velocity of 75 mi/hr in the opposite direction. The velocity of the ball relative to the bat is given by:

$$\mathbf{v} = \mathbf{v}_{BALL} - \mathbf{v}_{BAT}$$
$$\mathbf{v} = (-90 \text{ mi/hr}) - (+75 \text{ mi/hr})$$
$$\mathbf{v} = -165 \text{ mi/hr}$$




## 5.5 Components of a Vector

Addition and subtraction of vectors are algebraic operations that create one resultant vector out of two vectors. It is also possible to perform the reverse operation – form two vectors out of a single vector. Usually, these two new vectors are chosen to be at right angles to each other so that they form the sides of a right triangle with the original vector as the hypotenuse. We do this sometimes to make analysis of a problem easier to analyze.

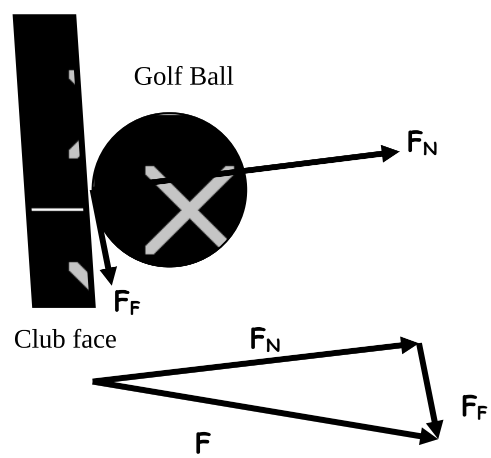



The projection of the vector $\mathbf{A}$ onto the x-axis is called the "x-component of $\mathbf{A}$," or $\mathbf{A}_X$. The projection of the vector $\mathbf{A}$ onto the y-axis is called the "y-component of $\mathbf{A}$," or $\mathbf{A}_Y$. The magnitudes of the components are related to the magnitude of the complete vector by the Pythagorean theorem:

$$A^2 = A_X^2 + A_Y^2$$

**Example – Contact Forces on a Golf Ball**

During contact the golf club exerts a contact force $\mathbf{F}$ on the ball. It is useful to resolve $\mathbf{F}$ into two components. The component $\mathbf{F}_N$ is normal (perpendicular) to the face of the club. The component $\mathbf{F}_F$ is caused by friction between the ball and the club face.



## 5.6 Problems to do:

1. In part, a vector is defined by its magnitude. How is a vector's magnitude specified?

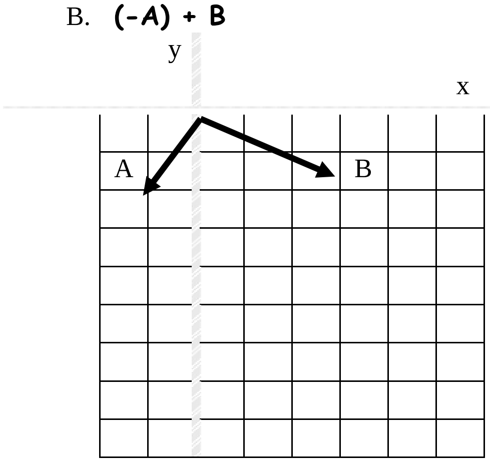
2. How is the direction of a vector defined?

3. How is a scalar defined?

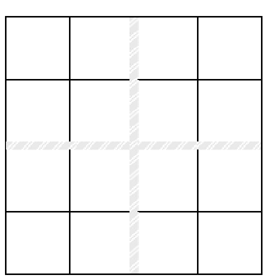
4. Two vectors added together equal zero. What conclusion can you draw?

5. When three or more vectors added together equal zero, what geometrical conclusions can be drawn? [HINT: sketch the figure.]

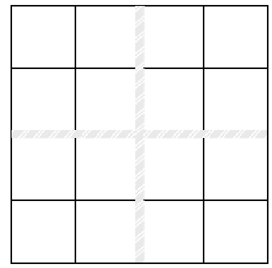
6. The vectors A and B are drawn from the origin. On the same graph draw the vectors:
   A. **A + B**
   B. **(-A) + B**




7. Given are the vectors $\mathbf{v_1}$ = +2 m/s in the x-direction and $\mathbf{v_2}$ = +1 m/s in the y-direction. On the diagrams below draw each of the following with the tails on the origin:
   A. $\mathbf{v_1}$ B. $\mathbf{v_2}$
   C. $-\mathbf{v_1}$ D. $-\mathbf{v_2}$



   E. $\mathbf{v_1} + \mathbf{v_2}$ F. $-\mathbf{v_1} - \mathbf{v_2}$
   G. $\mathbf{v_1} - \mathbf{v_2}$ H. $-\mathbf{v_1} + \mathbf{v_2}$



8. A force of 5 N east is added to a force of 4 N whose direction must be inferred from the sum. For each sum give the direction of the 4 N force:
   A. 9 N
   B. 1 N
   C. $\sqrt{41}$ N

9. The vector sum of four forces equals zero. Three of the forces are: (1) 3 N east, (2) 4 N west, and (3) 5 N east. What is the fourth force?

10. Given are two vectors **A** = 30 mi/hr east and **B** = 20 mi/hr at 20$^o$ north of east. Construct the parallelogram obtained by forming **A+B** and **B+A**.
    A. On the same construction what interpretation can be given to **+A+B** and **–A+B**?
    B. Under what circumstances will the magnitude of the two vectors (**+A+B** and **–A+B**) be equal?

11. What geometrical conclusion can be drawn if the vector sum of three vectors of equal magnitude equals zero?

12. A vector of magnitude 6 m consists of one component 3 m north and a second component directly east. What is the magnitude of the second component?

13. A jogger runs 4 km east and then 3 km north. Determine the jogger's vector displacement.

14. A velocity $\mathbf{v_1}$ has an x-component of +2 m/s and a y-component of +3 m/s, whereas the velocity $\mathbf{v_2}$ has an x-component of –2 m/s and a y-component equal to zero.
    A. Sketch each velocity vector.
    B. Calculate the magnitude of $\mathbf{v_1} + \mathbf{v_2}$
    C. Calculate the magnitude of $\mathbf{v_1} - \mathbf{v_2}$

# 6. Balls in Flight and Springboard Divers

**Projectile Motion**

## 6.1 Independence of Components of Motion

Our definitions of velocity and acceleration are the vector equations:

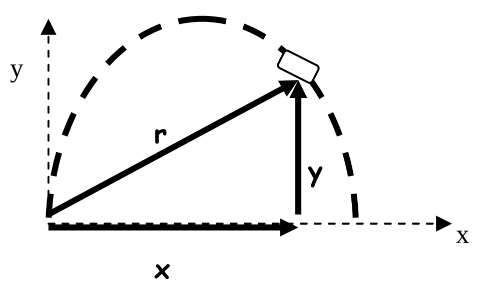
$$\mathbf{v} = \Delta\mathbf{r} / \Delta t$$
$$\mathbf{a} = \Delta\mathbf{v} / \Delta t$$

The vector **r** represents the displacement in the **xy**-plane, not just **x** or just **y**. The components of **r** are **x** and **y**. Similarly, the vector **v** represents the velocity in the **xy**-plane. The components of **v** are $v_X$ and $v_Y$. Finally, the vector **a** represents the acceleration in the **xy**-plane. The components of **a** are $a_X$ and $a_Y$. This is summarized in the table below:

| | | | |
|---|---|---|---|
| **Displacement** | r | x | y |
| **Velocity** | v | $v_X$ | $v_Y$ |
| **Acceleration** | a | $a_X$ | $a_Y$ |

Below is a picture of a possible path of a football thrown in the xy-plane. If I could freeze time at some instant (shown by the cute little football in my diagram below) the vector **r** is drawn from the origin to location of the football. The components of **r** are drawn "tail to tip" to illustrate the vector relationship

$$\mathbf{r} = \mathbf{x} + \mathbf{y}$$



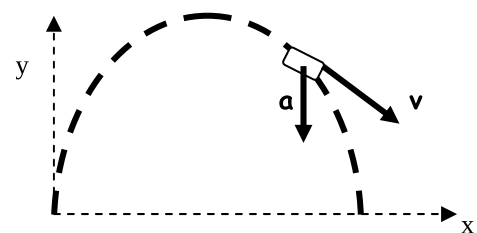

The path of the football in the **xy**-plane is shown by the dotted line. The velocity vector must be directed tangent to the dotted line at any instant, since the dotted line shows its motion. The velocity is always changing its direction because the football is always accelerating.

We are particularly interested in the motion of objects in a vertical plane under the influence of gravity (such as our football flying through the air!). Neglecting the effects of air resistance the acceleration is directed downward and equal to $\mathbf{a} = \mathbf{g} = 9.8\ m/s^2$. If we choose "up" to be the positive y-direction the acceleration components are:

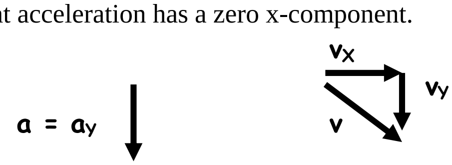
$$a_X = 0 \quad \text{and} \quad a_Y = -9.8\ m/s^2$$

for all objects.

For the instant chosen, the velocity and acceleration of the football is drawn on our diagram:




The basic relationship between velocity and acceleration vectors are always maintained for motion in a vertical plane (except that the angle or direction of the velocity is always changing). The components are shown below. Don't forget that acceleration has a zero x-component.

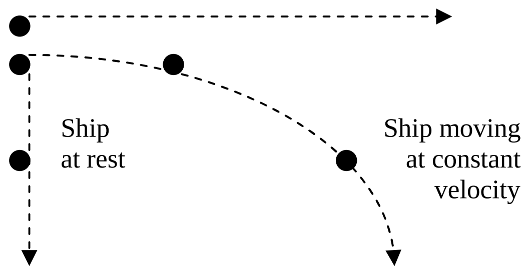



A remarkable property of this two-dimensional motion is that the components of the motion are independent of each other. The vertical motion is the same whether the object moves horizontally or not. Similarly, the horizontal motion is the same whether there is vertical motion or not. Galileo is credited with this discovery, and illustrated the idea with a sailing ship.

**Example – Galileo's Sailing Ship**

A ship is at rest in the harbor when a sailor drops a coin from a point on the mast. The path of the falling coin is a straight line down, parallel to the mast. This trajectory coincides with the y-axis shown below. Observers standing on the shore agree that the path is a straight line.

Suppose, however, that the coin is dropped from the mast while the ship sails along at constant velocity. Observers on the ship share the same horizontal motion as the coin, and again see its path of motion as being straight down and parallel to the mast. Observers on the shore do not share the ship's horizontal motion, and they see the coin's trajectory as a curve – falling downward but also moving sideways – although the distance between the mast and the coin remain fixed throughout the fall.




We may summarize the description of ship observers as follows:

(1) The trajectory of the falling coin is the same relative to the ship whether the ship is at rest or moving with constant velocity.
(2) The vertical motion of the falling ball is the same whether the ship is at rest or moving with constant velocity.

The shore observers witness quite different trajectories. The coin falls straight down from the resting ship, but travels along a curved path on the moving ship. Yet a sailor could not use the vertical motion of a falling ball to determine whether the ship was moving or at rest because that motion is not affected by horizontal motion.

This concept is called "Relativity" and it gave Einstein fits when he tried to apply it to light, but that is another story….

**Example – Balls on the Bus**

A student riding the bus on the way to a tennis tournament is a little nervous, so she tosses a tennis ball into the air and catches it. The bus is moving at 55 mi/hr down the road. She notes that the tennis ball does not blast through the rear window at 55 mi/hr, so it must be moving along with her. She sees the ball going straight up and then straight down, but a hitchhiker on the side of the road would have seen the ball moving in a large curved path because of its sideways motion. They would, however, agree on its vertical motion at all times.

The independence of components of the motion simplifies our study of two-dimensional motion. Instead of considering both directions simultaneously, we can look at them one at a time.

## 6.2 Time of Flight

The time required to fall does not depend upon horizontal motion. The falling coin reaches the deck of the ship in the same time whether the ship is at rest or in motion. To determine what the falling time depends on, lets go back to one of our TNEOM from chapter 3:

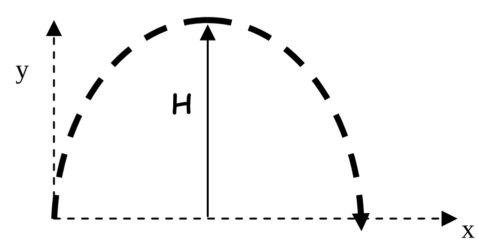
$$\Delta x = v_o * \Delta t + \tfrac{1}{2} * a * \Delta t^2$$

Let the initial velocity by zero, and the acceleration be gravity downward. We get:

$$\Delta t = \sqrt{(2 * \Delta y / a)}$$

The independence of components means that this time result holds whether an object is dropped or projected horizontally. Any object launched from ground level rises to a maximum height **H** and falls back

to the ground along a curved path shown below:



The path is symmetric about its peak, as we discussed back in Chapter 3. The falling time equals the rising time. The falling time is given by:

$$\Delta t_{FALL} = \sqrt{(2*\Delta y / a)}$$
$$\Delta t_{FALL} = \sqrt{(2*H / 9.8)}$$
$$\Delta t_{FALL} = 0.452*\sqrt{H}$$

This means that the total time of flight $t_F$ is:

$$t_F = 0.903*\sqrt{H}$$

Or the maximum height is given by:

$$H = 1.23*t_F^2$$

We see that the maximum height alone determines the time of flight.

## 6.3 Hang Time of a Punt

Given the hang time, the time of flight, we can calculate the maximum height of a punt. Broadcasters quote hang times during pro football games, and sometimes a little stopwatch is pictured on the screen so that viewers can see the hang times as well. In football, hang time is important because it gives the punting team's players time to get downfield so that they can tackle the punt receiver. Times typically range from 3.5 sec (poor) to 5.0 sec (excellent). Let's calculate the corresponding heights.

For $t_F$ = 3.5 s, $H = 1.23*(3.5)^2 = 15.1$ m.
For $t_F$ = 5.0 s, $H = 1.23*(5.0)^2 = 30.8$ m.

Because 30.8 m is about 101 feet, a punt having a 5.0 sec hang time could clear rather tall trees. The maximum height of the roof of the Houston Astrodome is 208 feet, so there is no chance that a punt could hit this roof.

In the New Orleans Superdome and instant replay screen facility was placed near the 50-yard line, and low enough so that fans could enjoy instant replays. It was situated 90 feet above the field, and therefore was in reach of a good punter. Ray Guy of the Oakland Raiders (the only punter ever taken in the first round of the draft, and the punter during the days when Kenny "**Snake**" Stabler quarterbacked the Raiders) punted a football that hit this facility causing the management to raise the height of the device.

Let's calculate the hang time for a 90 foot punt. First we need to convert the height to find that 90 ft = 27.4 m.

$$t_F = 0.903*\sqrt{H}$$
$$t_F = 0.903*\sqrt{(27.3)}$$
$$t_F = 4.73 \text{ s}$$

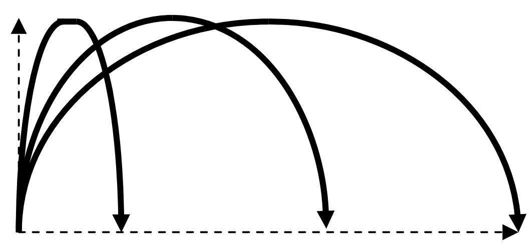
Presumably any punt kicked at the right spot with a hang time greater than 4.73 s could also have hit the instant replay facility.

In the astrodome, Hall of Fame third baseman Mike Schmidt struck a baseball that hit the roof. What wold be the hang time for a ball with a maximum height equal to the height of the Astrodome (208 ft or 63.4 m)?

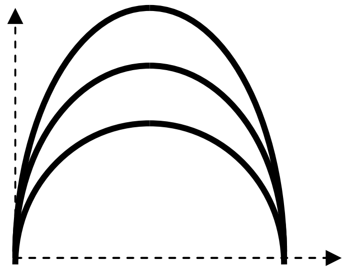
$$t_F = 0.903*\sqrt{H}$$
$$t_F = 0.903*\sqrt{(63.4)}$$
$$t_F = 7.19 \text{ s}$$

We have neglected air resistance in this calculation, and for a baseball moving this high air resistance would be important. Therefore the result must be taken as an estimate.

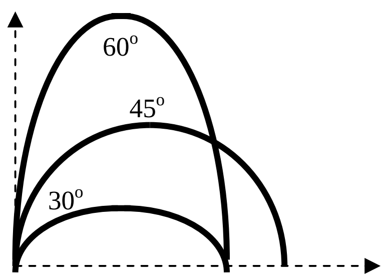
All trajectories reaching the same height correspond to the same time of flight. Footballs (and other similar objects) traveling farthest, for a given maximum height, started with greatest speed. Their vertical motion is the same, but they differ in their horizontal motion as shown below:



Now let's look at a picture of footballs traveling the same horizontal distance, but with various heights. Since the maximum heights differ, we know that the times of flight must differ as well. The greater the maximum height, the greater the time of flight.



Now let's look at a picture of footballs traveling with the same launch speed, but thrown at different angles. Suppose we were to draw trajectories for 30°, 45°, and 60° angles. We would discover that the football thrown at 45° went the farthest. We would also discover that the footballs thrown at 30° and 60° went the same distance – the 60° ball had a slower x-velocity but was in the air longer, while the 30° ball had a faster x-velocity but wasn't in the air as long.




## 6.4 One Meter Springboard Dive

The motion of a diver while airborne may be separated into two parts. Relative to her own center of gravity, the diver executes somersaults and twists. In addition to these motions the spectators observe the center of gravity of the diver to move in the same curved path as any other airborne object. Here we are concerned with the overall motion of the diver. Later, we may get to consider the diver's rotation.

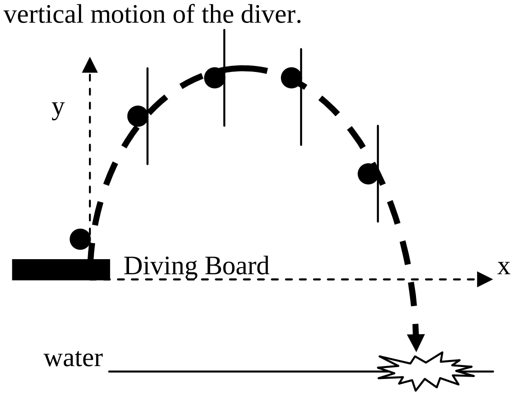
We start by treating the diver as a point-sized object located at, and moving with, her center of gravity. Let's consider the diver's time of flight and determine its effect on the difficulty of the dive.

In treating the motion of the diver in a plane we take advantage of the independence of components of motion to consider the horizontal and vertical motions separately. The horizontal motion is simplified because the horizontal acceleration equals zero. Thus, we have:

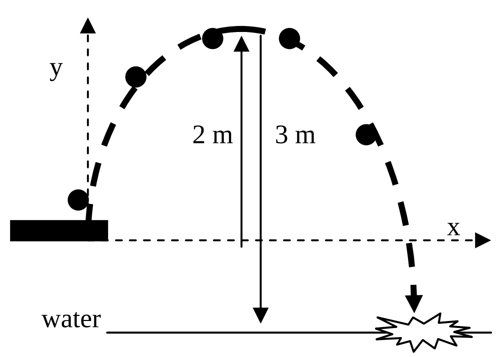
$$v_X = v_o = \text{constant}, \text{ and } a_X = 0$$

$$\text{so} \quad \Delta x = v_o * \Delta t + \tfrac{1}{2} * a_X * \Delta t^2 = v_X * \Delta t + 0$$

The horizontal distance from the diving board increases linearly with time. The trajectory is shown below, and equal time intervals are marked. Equal horizontal distances are covered in equal times. This fact is not changed by the vertical motion of the diver.




Similarly, the vertical motion is not affected by the horizontal motion. We can just ignore the horizontal motion while solving the vertical part of the problem. We assume the diver's center of gravity starts about 1 m above the diving board (roughly ½ the height of a person) and 2 m above the water, when she stands on the end of the board. The spring in the board allows the diver to raise her center of gravity about 2 m above its starting elevation, or to a point about 4 m above the water. We define the dive to be over when the center of gravity has fallen to about 1 m of the water's surface (when part of the body starts to enter the water). Thus, the diver's center of gravity rises 2 m and falls 3 m during the dive.




The time of flight equals the time to rise plus the time to fall, but we note that these times are not the same because the dive is not symmetrical. This means that we need to calculate the two times separately:

The time to rise 2 m is the same as the time to fall 2m. I chose "up" to be positive:

$$\Delta y = v_{oy}*\Delta t_{RISE} + \tfrac{1}{2}*a_y*\Delta t_{RISE}^2$$
$$-2.0\text{ m} = 0 + ½*(-9.8\text{ m/s2})*\Delta t_{RISE}^2$$
$$\Delta t_{RISE} = 0.639\text{ s}$$

from the board to the top of the dive. The time to fall is just as quick to calculate:

$$\Delta y = v_{oy}*\Delta t_{FALL} + \tfrac{1}{2}*a_X*\Delta t_{FALL}^2$$
$$-3.0\text{ m} = 0 + ½*(-9.8\text{ m/s2})*\Delta t_{FALL}^2$$
$$\Delta t_{FALL} = 0.782\text{ s}$$

The total time of flight is therefore 1.42 seconds.

## 6.5 Three Meter Springboard Dive

The rising part of the 3 m springboard dive is identical to the rising part of the 1 m springboard dive. The center of gravity rises 2 m to a height of 6 m above the water. The time to the high spot in the dive is 0.639 s, just as before.

The time to fall is just like before:

$$\Delta y = v_{oy}*\Delta t_{FALL} + \tfrac{1}{2}*a_X*\Delta t_{FALL}^2$$
$$-6.0\text{ m} = 0 + ½*(-9.8\text{ m/s2})*\Delta t_{FALL}^2$$
$$\Delta t_{FALL} = 0..010\text{ s}$$

The total time of flight is therefore 1.65 seconds.

## 6.6 Ten Meter Platform Dive

For the 10 m platform dive we assume that the diver does not elevate her center of gravity, but instead simply falls off the platform and into the water below. This means that there is no time of rise to consider, but only a time of fall.

$$\Delta y = v_{oy}*\Delta t_{FALL} + \tfrac{1}{2}*a_X*\Delta t_{FALL}^2$$
$$-10.0\text{ m} = 0 + ½*(-9.8\text{ m/s2})*\Delta t_{FALL}^2$$
$$\Delta t_{FALL} = 1.43\text{ s}$$

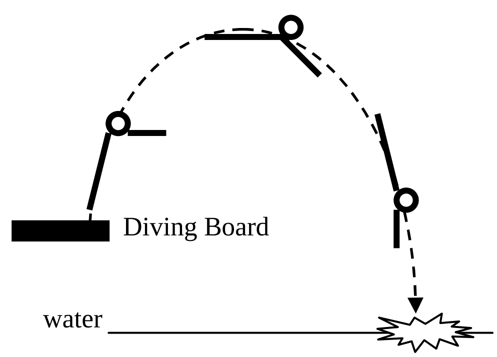
The three times of flight we calculated are the times available to the diver to execute her dive. We found roughly equal times for the execution of 1 m springboard and 10 m platform dives, but a significantly larger time for the 3 m springboard dive. To the extent that the difficulty of the dive depends on the execution time, a given dive should be easier off the 3 m board than off either the 1 m or 10 m boards.

Difficulties are shown below for several selected dives. In no case is the 3 m difficulty higher than either of the other two. The calculations are approximate and sensitive to the elevation of the center of gravity that is assumed.

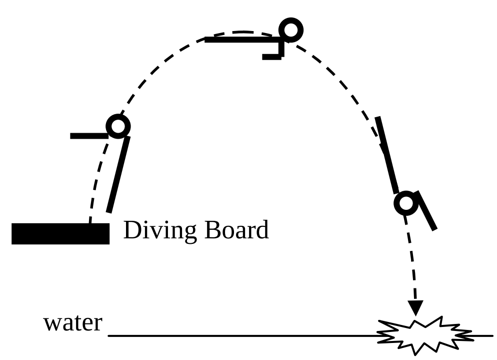
| | 1 m | 3 m | platform |
|---|---|---|---|
| 1 ½ Pike (front) | 1.9 | 1.8 | 1.8 |
| 1 Tuck (back) | 1.7 | 1.6 | 1.7 |
| 1 Tuck (reverse) | 1.7 | 1.6 | 1.7 |
| 1 ½ Straight (front) | 2.9 | 2.8 | 2.8 |

Dives may be launched in four ways that are shown below. Clearly the difficulty also depends upon the type of launch. Our analysis only takes into account the available execution time.

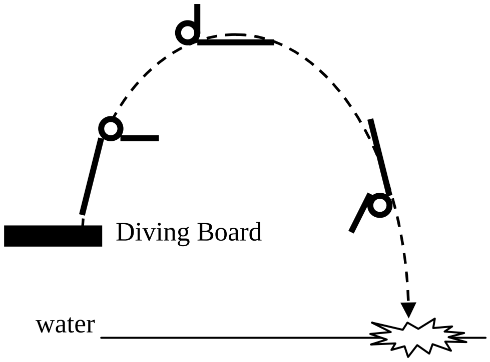
**1. Forward dive** –



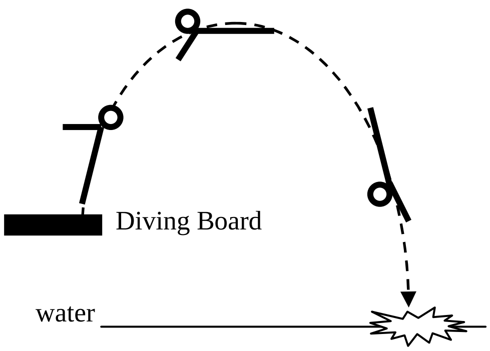

**2. Backward dive** –



**3. Reverse dive** –

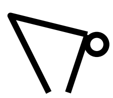



**4. Inward dive** –

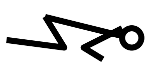



There are also different positions from which a diver can do the spins and flips needed in the dive.

**Pike position** –



**Tuck position** –



**Layout position** –

## 6.7 Problems to do:

NOTE: Assume that air resistance can be neglected unless specifically stated otherwise.

1. In the diving analysis, explain why vertical motion could be treated as if there was no horizontal motion.

2. For each of the quantities tell whether or not they affect the time of flight:
   A. horizontal distance
   B. vertical distance
   C. horizontal velocity
   D. vertical velocity

3. What two quantities are needed to calculate the time of flight for a projectile?

4. How long is a trampoline artist airborne who elevates his center of gravity 5 m during a jump?

5. A baseball is observed to rise 45 m. What was its time of flight?

6. A baseball has a time of flight of 7 s. What is its maximum height?

7. A quarterback wishes to throw a pass to a receiver downfield. What kind of trajectory will minimize the time of flight?

8. A football kicked at $65^o$ with the horizontal lands at its launch elevation. With the same launch speed what other launch angle would cause a landing in the same point?

9. A football is kicked at $60^o$ and later at $40^o$. The two kicks start with the same speed.
   A. Which if the kicks travels farther?
   B. Which of the kicks travels higher?
   C. Which of the kicks is airborne longest?

10. Compare the horizontal distances covered by a kicked soccer ball rising and falling. Assume launch and landing elevations are the same.

11. If a series of punts all have approximately equal hang times, what conclusion could be drawn?

12. A punt by Ray Guy struck an instant replay device suspended 90 feet above the field in the New Orleans Superdome. What maximum time elapsed between the punt and the impact?

13. What height above the water would a board have to be in order for a diver to be airborne for 2 s, if the type of board is:
    A. a springboard
    B. a platform diving board

14. A springboard diver strikes the water at 7 m/s. From what height did she fall?

15. How long would a springboard diver be airborne if her center of gravity was elevated 1 m by the spring:
    A. for the 1 m springboard.
    B. for the 3 m springboard.

16. When a diver faces the water, but rotates toward (rather than away from) the diving board, the dive is given what label?

17. Would you expect a triple somersault to be easier on the 3 m board or 1 m board? Explain.

18. According to execution-time considerations, which of the three diving board events should be most difficult for a particular dive? Explain.

19. Explain why divers spend the same time above the one and three meter boards if executing the same dive.

20. In a three-meter springboard dive what fraction of the total airborne time does the diver spend between the diving board and the water? (Assume a 2 m elevation of the center of gravity due to the spring.)