2. Runners and Sky Divers

Straight Line Acceleration

2.1 Pace in Running:

Average velocity determines the victor in a race. The participant having the largest average velocity will win. To solve this, we re-arrange the equation $v_{AVG} = \Delta x / \Delta t$ and get:

$$\Delta t = \Delta x / v_{AVG}$$

For a race over a fixed distance Δx is constant. The first performer to cross the finish line has the smallest Δt and wins the race. The equation shows that for a given fixed Δx, the largest v_{AVG} will correspond to the smallest Δt. Hence, the winning runner has the largest average velocity.

Example – A 400 meter Race

In a 400 m race runner A has an average velocity of 7 m/s and runner B has an average velocity of 6 m/s. What are their times and which runner wins? First we calculate the time for runner A.

v_{AVG} = 7 m/s and Δx = 400 m
so $\Delta t = \Delta x / v_{AVG}$ = (400 m) / (7 m/s) = 57.1 s

Similarly for runner B we obtain:

v_{AVG} = 6 m/s and Δx = 400 m
so $\Delta t = \Delta x / v_{AVG}$ = (400 m) / (6 m/s) = 66.6 s

Runer A wins because she has the smaller time.

Distance and time are readily available in a race, but average velocity must be calculated. For this reason runners keep track of their time as a measure of pace. For a 400 m race, a time of 100 s is a 4 m/s pace, whereas a time of 50 s corresponds to a 8 m/s pace. Thus, improvement in the runner's performance accompanies an increase in average velocity and a decrease in the time.

The average velocity which a given runner achieves in a race depends on the length of the race. For a given runner the longer the race the smaller the average velocity will be. As an indication of this effect consider the world's records (1974 data) for male runners given in the table below: The velocities range from 10.1 m/s at 100 meters to 5.46 m/s for a 30 kilometer run.

Running records (1974 data)

x (m)	t (s)	v (m/s)	time per mile (s)
100	9.9	10.10	159
200	19.8	10.10	159
400	43.8	9.13	176
800	103.7	7.71	209
1,500	212.2	7.07	228
5,000	793.0	6.31	256
10,000	1650.8	6.06	266
20,000	3464.4	5.77	279
30,000	5490.4	5.46	295

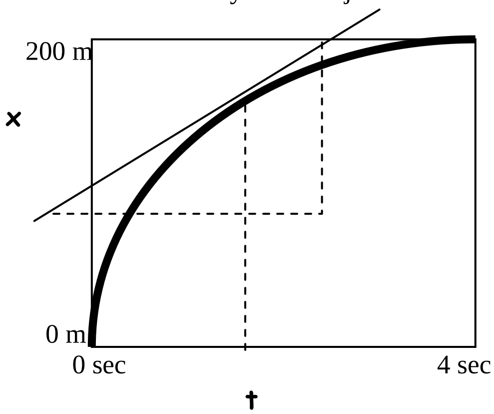
The reciprocal relation between pace and time per lap or time per mile is brought out by looking at the 3rd and 4th columns of the data. It is common to use the time per mile (or time per km) as a measure of pace for runners. Note that with increasing distance, the pace decreases and the time per mile increases.

2.2 Instantaneous Velocity:

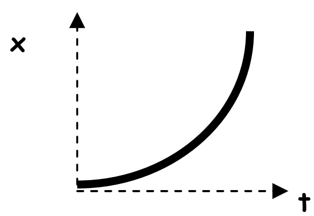
Average velocity is calculated during a time interval. The time interval may be large or small, but two times are required. These times are the initial and final times for the time interval.

As an object moves, however, it has a velocity at each moment in time, and not just during time intervals. The speedometer on a car exhibits the velocity of the car at every moment. How can we define the velocity of an object at one instant? The actual mathematical method requires calculus, but we shall get a decent answer using an $x-t$ graph and a tangent line.

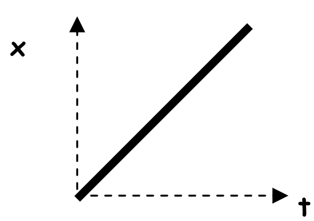
A straight edge is placed tangent to the curve at the desired point, and the tangent line is drawn. The slope of the tangent line is obtained by choosing an arbitrary two points on the tangent line and finding the slope between those two points. The slope of the tangent line represents the slope of the curve at that instant, or the instantaneous velocity of the object!



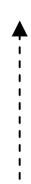
Note that the above graph shows an object that starts out moving fast (steep graph or big slope) but slows down later on (flatter graph or small slope). Let's look at a few examples of how an x-t graph can be interpreted. What does this graph represent?

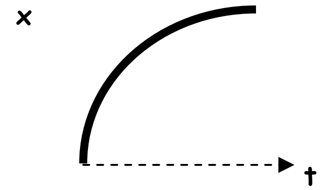


It's a case of an object that starts out slow at first (flat slope), but gets faster later on (steep slope). This could be a sprinter coming off the block, or a football receiver leaving the line of scrimmage when the ball is snapped. What is the next graph?



It's an object in motion at constant speed (since a line has constant slope). This could be a bowling ball rolling down a lane, or a marathon runner somewhere around mile #13. How about the third graph below?



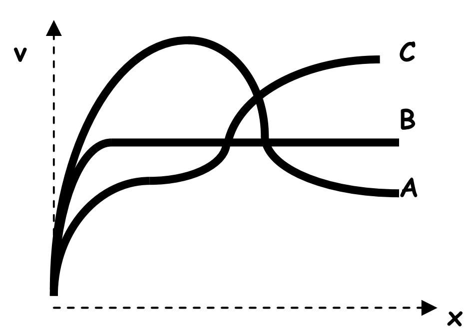


We saw this one before. It's an object slowing down. This might be a sprinter just after she crosses the finish line. Note that the slope is positive for all three cases we just discussed. This means that the velocity is positive for all three cases as well – none of these objects are moving backwards! We saw that the average velocity gives an overall impression of pace. The average velocity for an entire race gives no information, however, regarding the variation of pace within a race. This variation can only be expressed using instantaneous velocity.

Psychology plays an important part in running. When Herb McKenly set a world's record in the 440 (yard) dash, he ran as fast as he could initially, and then struggled to finish. His hope was to establish an insurmountable lead in the first part of the race. His tactic (illustrated by curve **A** below) was to discourage the competition early in the race.

During the 1972 Olympic marathon, Frank Shorter set a rapid and constant race for the entire race. He budgeted his energy so that it was expended uniformly. His tactic earned the gold medal, and is illustrated by curve **B** below.

Miruts Yifter depends on his blazing speed at the finish. For this tactic to work, he must stay close to the leader until the last 400 meters or so of the race. At this point he begins to sprint. Using this approach, Yifter won the 5,000 and 10,000 meter races in the 1980 Olympics. This running pattern is illustrated by the curve labeled ***C***. Watch out: this is a **v-x** graph, not a **x-t**.



To summarize these three tactics illustrated above, we might say Runner **A** [McKenly] starts fast and then fades, Runner **B** [Shorter] runs at a steady pace, and Runner **C** [Yifter] saves himself for a fast finish.

2.3 Average Acceleration

To introduce the last of our kinematic quantities – acceleration – we follow the same procedure used with velocity. First we consider the special case where the acceleration is constant, and then extend our treatment to include a variable acceleration.

Acceleration refers to the time rate of change of velocity. To get a grip on the concept, imagine you are looking at the speedometer in a car travelling in a straight line. If the needle is moving, the car is accelerating. If the needle is stationary, the car is not accelerating.

Average acceleration is defined analogously to average velocity. The defining equation is:

$$a_{AVG} = {}^{\Delta v} / {}_{\Delta t}$$

where Δv is the change in velocity and Δt is the time interval. Units for acceleration are determined by the defining equation. Because we know that the units for Δv are m/sec and that the units for Δt are sec, we infer from the equation that units for acceleration are m/sec^2.

Treat units like variables in algebra. Divide m/sec by sec/1– division of two fractions means we invert the second fraction and multiply – so we multiply m/sec by ^{1}/sec to get m/sec^2.

2.4 Constant Acceleration

Cases where a_{AVG} remains constant are important and will be developed in special equations later on. A special example of constant acceleration is that of gravity near the surface of the Earth. The actual value of this acceleration is based on altitude and latitude, but we can use

$$a_{GRAVITY} = g = 9.80 \text{ m/s}^2$$

as a general value. Note that the acceleration of gravity is so special that we give it its own symbol: g. Most times something falls because of gravity, we can use g as its acceleration.

Example – Velocity Acquired in Freefall

Starting with zero initial velocity, a sky diver falls freely for 3 seconds. We wish to determine the diver's velocity change during this time interval. Using $a_{AVG} = g = 9.80$ m/s^2 and $t = 3$ sec, we can calculate:

$$a_{AVG} = \Delta v / \Delta t \quad \text{so} \quad \Delta v = a_{AVG}\, \Delta t$$
$$\Delta v = (9.80 \text{ m/s}^2)*(3 \text{ s}) = 29.4 \text{ m/s}$$

For comparison, 29.4 m/s is 65.8 mi/hr. It doesn't take long for the velocity to build up! Air resistance (not considered here) is important at high velocities, and would increase the time required to actually reach this speed.

Note that we re-wrote our average acceleration equation in order to solve the above example for Δv. We could also re-write the same equation to solve for Δt if we needed.

Example – Time to Acquire a Velocity

The velocity of a falling object increases from zero to 100 mi/hr (44.7 m/s). Neglecting air resistance, how long does it fall?

$$\Delta v = (44.7 \text{ m/s}) – (0 \text{ m/s}) = 44.7 \text{ m/s}$$
$$a_{AVG} = \Delta v / \Delta t \quad \text{so} \quad \Delta t = \Delta v / a_{AVG}$$
$$\Delta t = (44.7 \text{ m/s}) / (9.80 \text{ m/s}^2) = 4.56 \text{ s}$$

Therefore, it takes 4.56 s for this object to fall from rest to 100 mi/hr.

Example – Acceleration of a Drag Racer

A drag racer requires 8 s to reach a speed of 60 mi/hr (26.7 m/s) starting from rest. What is the average acceleration?

$$a_{AVG} = \Delta v / \Delta t = (26.7 \text{ m/s} – 0 \text{ m/s}) / (8 \text{ s})$$
$$a_{AVG} = (26.7 \text{ m/s}) / (8 \text{ s}) = 3.35 \text{ m/s}^2$$

The drag racer undergoes an average

acceleration of about a third the acceleration of gravity.

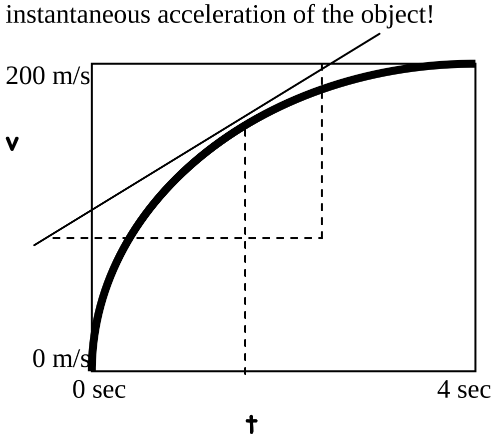
Now we consider a graph in which velocity is plotted against time. (Time is always on the horizontal axis!) This graph must not be confused with the **x-t** and **v-x** graphs we have already seen: this is a **v-t** graph. For motion with constant acceleration, the v-t graph is a straight line. On a v-t graph, we note that Δv is the "rise" and that Δt is the "run." Thus:

$$a_{AVG} = \Delta v / \Delta t = \text{slope of a v-t graph!}$$

We find another case where the slope of a graph has an actual physical meaning!

2.5 Instantaneous Acceleration

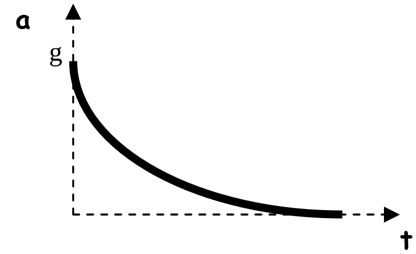
To define instantaneous acceleration we follow the same procedure we used with instantaneous velocity. Thus, the instantaneous acceleration is defined to be the slope of the tangent line to the **v-t** graph at a point. To obtain this acceleration from a graph, we draw a line tangent to the curve at the point in question and find the slope of that line. That slope would represent the instantaneous acceleration of the object!



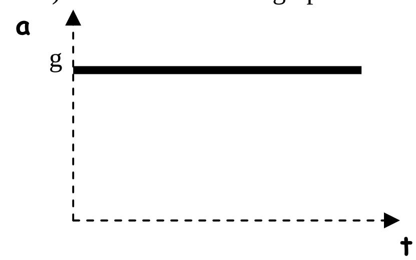
2.6 At Last: Sky Divers!

Consider a sky diver falling through the air. At the instant the sky diver leaves the plane, he falls like any other object. His initial acceleration equals the acceleration of gravity. Air resistance is so small at low velocities that it can be neglected initially. He falls like an object falls in vacuum.

After falling for some distance, the sky diver's velocity increases. As his velocity increases, air resistance becomes more and more important. The effect of air resistance is to reduce his acceleration. His acceleration is gradually reduced as his velocity increases until his acceleration falls to zero (and his velocity becomes constant). Initially **a=g** and **v=0**, but much later **a=0** and **v=**constant. The behavior of this acceleration is shown in this graph:



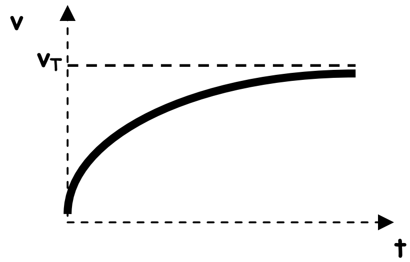
For comparison the acceleration of an object in free fall with no air resistance (such as in a vacuum) is illustrated in the graph below:



Initially, the sky diver has zero velocity but a large acceleration (9.80 m/s^2). As he falls the velocity increases to a maximum value, called the "terminal velocity" v_T, while the acceleration falls to zero. Typical values for v_T are 100 mi/hr (sky diver in spread-eagle position) and 200 mi/hr (sky diver in nosedive position).

The terminal velocity of a sky diver depends upon the diver's position and orientation. In the opening scene of the film **Moonraker**, James Bond catches a sky diver from behind by utilizing this difference in v_T. He, in turn, is caught by Jaws who uses the same technique.

The graph below shows velocity vs. time for a sky diver falling through the air, and have included the effect of air resistance.



To begin with the velocity is zero. At the same time the slope (and therefore the acceleration) is large. As time passes, the velocity increases and gradually approaches the terminal velocity v_T. Meanwhile, the slope and the acceleration gradually approach zero.

Problems to do:

1. Runner **A** completes 1500 meters at an average velocity of 7.30 m/s while runner **B** requires less time at a average velocity of 7.40 m/s. Determine the time for each runner and the time interval between the finishes. [Keep 5 figure accuracy in the calculation to achieve 3 figure accuracy in the time interval answer.]

2. Two students run on an indoor track requiring 10 laps to complete one mile. **A** has a 48 sec lap time and **B** has a 42 sec lap time. Calculate the two average velocities and the two times for the mile distance.

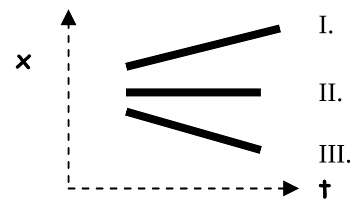
3. A runner completes 1 mile in 5 minutes. Calculate the time needed to run 6 miles at this pace in seconds, minutes, and hours.

4. A pitch travels 17 m from pitcher to batter after being released by the pitcher. Calculate the time required for the baseball to reach the batter traveling at the horizontal velocity:
   A. 100 mi/hr
   B. 80 mi/hr

5. Positions and times are given in the table for a rolling ball. Determine which one-second interval corresponds to the greatest velocity:

x (meters)	t (seconds)
0.0	0.0
1.0	1.0
2.0	2.0
4.0	3.0
4.0	4.0
3.0	5.0
1.0	6.0

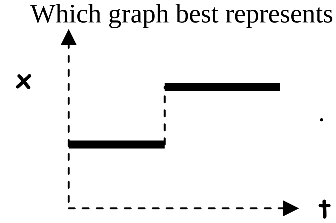
6. Three **x-t** graphs are shown below.



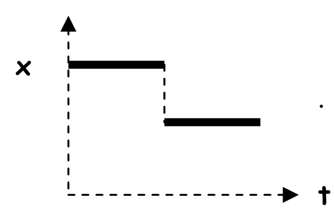
   Which graph represents the following:
   A. motion.
   B. rest.
   C. constant velocity.
   D. zero acceleration.
   E. zero velocity.

7. Determine which of the graphs from #6:
   A. have positive slope.
   B. have zero slope.
   C. have negative slope.

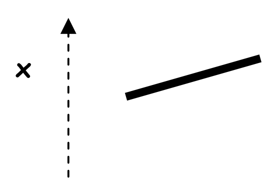
8. A runner runs rapidly for the first half and slowly for the second half of the race. Which graph best represents this tactic?



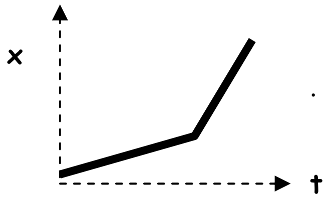
Graph I.



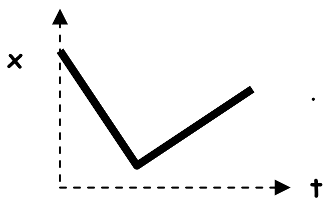
Graph II.



Graph III.

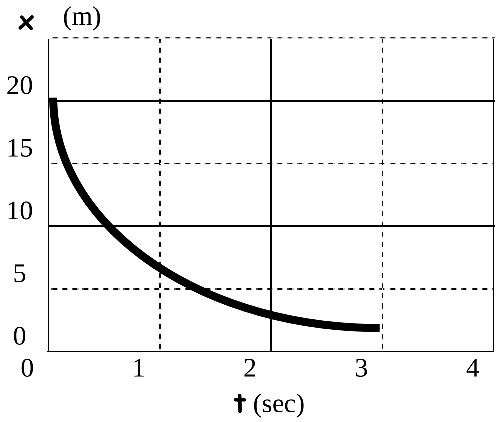


Graph IV.



Graph V.

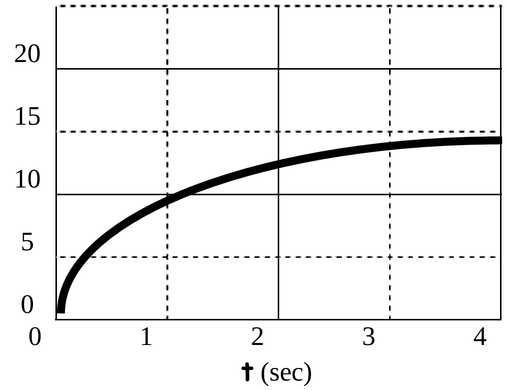
9. What is the average velocity for the object in the graph below between **t** = 1 s and **t** = 2 s?



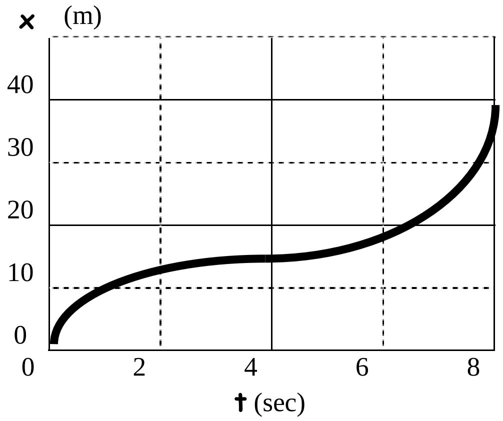
10. For the graph in #9, obtain the velocity at:
    A. **t** = 1.4 sec.
    B. **t** = 2.4 sec.
    C. **t** = 0.4 sec.

11. A runner moving at 5 m/s at **t** = 2 sec is moving at 11 m/s by **t** = 5 s. Calculate the runner's average acceleration between these two times.

12. For the graph below, determine the velocity at **t** = 1 sec.



13. A skier moves as shown. Calculate her velocity at **t** = 6 sec.



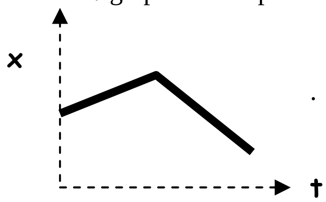
14. Look at the **v-t** data table below. What is the average acceleration between **t** = 1 s and **t** = 3 s?

v (m/s)	t (seconds)
0.0	0.0
3.0	1.0
12.0	2.0
27.0	3.0

15. A bicyclist is observed to have a velocity of 13 m/s at t = 2 sec and 19 m/s at t = 4 s. Calculate his average acceleration.

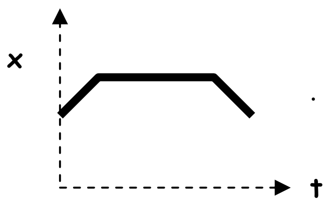
16. A runner with initial speed v_o increases her speed gradually during the first half of a race, and then decreases gradually back to v_o

during the second half of the race. Which of the **v-t** graphs best represents her race?

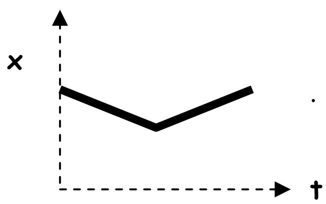


Graph I.

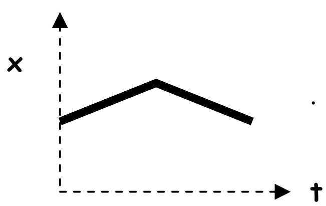
Graph II.



Graph III.



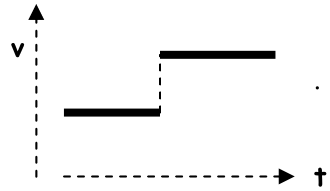
Graph IV.



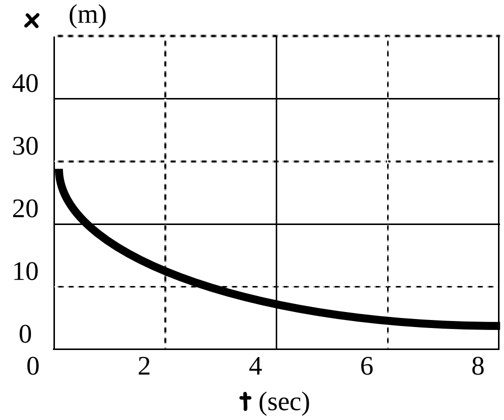
Graph V.

17. What property of a velocity vs. time graph corresponds to the acceleration?

18. The **v-t** graph below is based on one of the **x-t** graphs in #8. Which one?

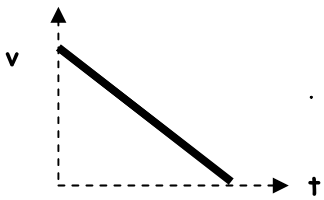


19. The velocity vs. time graph for a car is shown. What is the acceleration at **t** = 2 s?

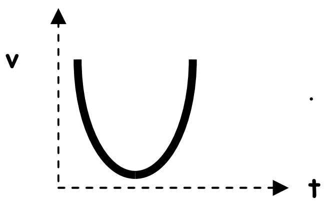


20. Which of the following graphs best represents this table of data?

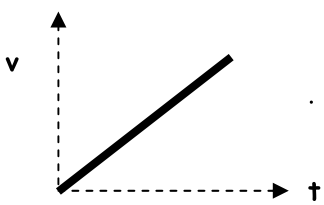
x (m)	v (m/s)	t (sec)
0.0	0.0	0.0
2.7	8.1	1.0
21.6	32.4	2.0
72.9	72.9	3.0



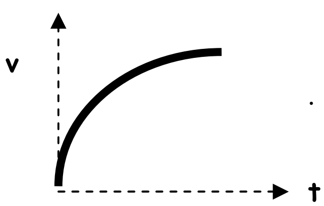
Graph I.



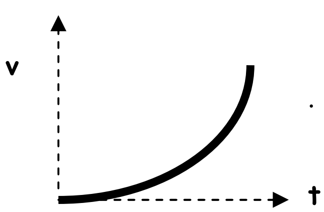
Graph II.



Graph III.



Graph IV.



21. Look at the table below. The actual position is unknown for t = 3 s. Use a graph to find the missing position.

x (meters)	t (seconds)
0.0	0.0
2.0	1.0
16.0	2.0
? ?	3.0
128.0	4.0

22. Look back to find the Running records data back in section 2.1 and determine the following:
   A. time to run 300 m.
   B. time to run 3,000 m.

23. One rule-of-thumb states that a runner able to run a certain distance in a time t would require the time 2.2t to run twice this distance. According to this rule a runner needing 44 min to run 10 km will require how long to run 20 km?

24. A jogger runs 2 miles in 16 min. What tie is required for the same jogger running 1 mile? 4 miles? (Use the same rule as in #23.)

25. A jogger runs 1 mile in 7 min. According to the rule in #23, what should be her time for 2 miles?

26. Construct a graph of velocity vs. time for an airborne volleyball player making a kill shot. Consider both the time rising and the time falling. From the slope of this graph construct the acceleration vs. time graph.

3. Vertical Jump and Hang Time

Motion with constant acceleration

3.1 Three Nifty Equations of Motion

If we look at the definitions of velocity and acceleration from before, we will find that there are five basic physical quantities contained within them (I have slightly changed notation for velocity here):

Δx = the displacement of the object
V_i = the object's initial velocity
V_f = the object's final velocity
a = the object's acceleration
t = the size of the time interval

By taking the definitions of velocity and acceleration and a little algebra, it is possible to combine them to create new forms of the same information that we already used. I call these the "three nifty equations of motion" (TNEOM) because they can be used in so many different types of motion problems. When in doubt, start with the TNEOM and see if they get you to the answer.

The TNEOM:

$$V_f = v_i + a*t$$
$$V^2 = v_i^2 + 2*a*\Delta x$$
$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$

Note that each equation contains four of the five basic quantities of motion. This means that if we have three pieces of given information we should be able to solve for the answer. Three equations. Three bits of info. Nifty, huh?

Memorize them. Love them. Tell your friends.

Example – A falling rock

You drop a rock from a bridge and hear it smite the water below 2 seconds later. What is the height of the bridge?

Simple. We just find three bits of information, and… uh… let's see. We have t but what are the other two? Here's a secret: one reason why people think Physics is so hard is that Physicists are lazy. If there is information that you can figure out for yourself then we don't always state it in the problem. It's there, but you have to hunt for it. So, what two pieces of data are hidden?

(1) See the word "drop?" I just told you that the initial velocity for the rock is zero. Always look for this sort of hidden data.

(2) What made the rock fall? Gravity! So $a = g$, and g= 9.80 m/s^2.

Okay, so we now know three things: t, v_i, and a. Look at the TNEOM and find the equation that contains our three plus the Δx that we are looking for. Then simply solve for Δx.

$$\Delta x = v_i*\Delta t + \tfrac{1}{2}*a*t^2$$
$$\Delta x = (0\text{ m/s})(2\text{ s}) + ½(9.8\text{ m/s}^2)(2\text{ s})^2$$
$$\Delta x = (0\text{m}) + (19.6\text{ m}) = 19.6\text{ m}$$

Thus, the bridge was 19.6 m tall.

Is it always that simple? Not quite. There may be a two-step problem or two objects in motion together, so use of several of the TNEOM might be needed to solve the problem.

Watch out: All of the TNEOM are created under the notion that acceleration is a constant. If acceleration changes, the TNEOM won't work!

3.2 The Mythical 4th Equation

Some people think that they have found a long-lost 4th Equation, which would turn the TNEOM into the FNEOM. Many textbooks contain this:

$$v_{AVG} = \tfrac{1}{2}(v_i + v)$$

Look at it again and you will see that this equation is just a mathematical average of velocity values, and not a Physics equation at all. Nice try, guys, but they're still the TNEOM!

3.3 Standing Vertical Jumps

One of the tests used to determine the explosive thrust of an athlete's legs is the standing vertical jump. The subject stands next to a wall, reaches as high as possible, and puts a horizontal chalk mark on the wall. Then the subject jumps as high as possible without taking any preliminary steps. At the peak of the jump the subject puts a second horizontal chalk mark on the wall. The vertical distance between the two chalk marks represents the elevation of the subject's center of gravity during the jump. This elevation, in turn, is a measure of the subject's explosive leg thrust.

Example – Student jumping

A typical male college student is able to jump about 20 inches, or 0.508 meters. Let's use this information to determine his launch velocity and time of flight. Choosing "up" to be the positive direction (and thus "down" to be negative) we are given that $\Delta \mathbf{x}$ = +0.508 m, $\mathbf{v}$ =0, and $\mathbf{a}$ = g = -9.80m/s^2. Notice that $\mathbf{v}$ represent the jumper's velocity at the peak of his jump. We set $\mathbf{v}$ = 0 because the jumper is motionless for an instant at the top of his jump. The acceleration is negative because gravity pulls down and we selected the direction down to be negative.

Search for one of the TNEOM that contains our three known quantities, plus $\mathbf{v}_i$ which is what we want to solve.

$$\mathbf{v}^2 = \mathbf{v}_i^2 + 2*\mathbf{a}*\Delta\mathbf{x}$$
$$\mathbf{v}_i^2 = \mathbf{v}^2 - 2*\mathbf{a}*\Delta\mathbf{x}$$
$$\mathbf{v}_i^2 = 0^2 - 2(-9.80\ \text{m/s}^2)(0.508\ \text{m})$$
$$\mathbf{v}_i^2 = +9.96\ \text{m}^2/\text{s}^2$$
$$\mathbf{v}_i = +3.16\ \text{m/s}$$

So we find that $\mathbf{v}_i$ = 3.16 m/s is that student's launch velocity. This is the velocity of the jumper's center of gravity at the moment he leaves the ground. Knowing $\mathbf{v}_i$ gives us four of the basic quantities of motion, so finding the time interval should be easy because a couple of the TNEOM may work now.

$$\mathbf{v} = \mathbf{v}_i + \mathbf{a}*\mathbf{t}$$
$$\mathbf{t} = (\mathbf{v} - \mathbf{v}_i) / \mathbf{a}$$
$$\mathbf{t} = (0\ \text{m/s} - 3.16\ \text{m/s}) / (-9.80\ \text{m/s}^2)$$
$$\mathbf{t} = +0.322\ \text{s}$$

The jumper requires 0.322 seconds to rise from the floor to the peak of his jump. Note how careful we had to be with plus and minus signs in these calculations. The minus sign used with the acceleration made the time positive as required.

Let's consider a similar problem using a different approach. The jumper rises to the top in a <u>rise time</u> and falls back in a <u>fall time</u>. The jump is symmetrical in time so that the time to rise and time to fall should be the same. (Why? Gravity slows him down on the way up and speeds him up on the way down. This happens over the same distance up and down. The speed at the top is zero – final for the way up but initial for the way down. It makes sense that the times would also be the same.) Let's use this time symmetry to solve a problem.

Example – Michael Jordan

Michael Jordan, a former pro basketball player and movie star, is credited with a 42 inch vertical jump. Let's determine his launch velocity and rise time for the jump by calculating the time to fall from the top of his path and using this result to infer what happened during the rise. To emphasize that the choice of which direction is positive (with the opposite being negative), and for convenience, we'll choose down as plus (so up is minus). We are given:

$$\Delta\mathbf{x} = +1.067\ \text{m},\ \mathbf{v}_i = 0,\ \mathbf{a} = +9.80\ \text{m/s}^2$$

In this case, $\Delta\mathbf{x}$ is positive because we are moving down. $\mathbf{v}_i$ = 0 because we are choosing to start our clock at the top of the path and study the motion on the way down. $\mathbf{a}$ is positive because gravity is still pulling down.

$$\Delta\mathbf{x} = \mathbf{v}_i*\mathbf{t} + \tfrac{1}{2}*\mathbf{a}*\mathbf{t}^2$$
$$1.067\ \text{m} = 0 + \tfrac{1}{2}(9.8\ \text{m/s}^2)\mathbf{t}^2$$
$$\mathbf{t}^2 = 0.218\ \text{s}^2$$

$t = 0.467$ s

By symmetry, the total time of flight is twice the fall time, giving an answer of 0.934 seconds. This is about 45% better than the number we calculated for the typical college male student.

Remember, we are also interested in MJ's launch velocity. It can be calculated quickly once we realize that by symmetry the launch velocity is the same magnitude as the landing velocity – the only difference is in the direction! Find the landing velocity:

$$v = v_i + a*t$$
$$v = 0 + (9.80 \text{ m/s2})(0.467 \text{ s})$$
$$v = +4.58 \text{ m/s}$$

or about 10.2 mi/hr. Because we chose down to be positive, the actual launch velocity is negative (–4.58 m/s).

3.4 Measurement Error

To measure the vertical jump, we could measure the time of flight and calculate the vertical distance, or we could measure the vertical distance and calculate the time of flight. It is not difficult to measure the vertical distance with an error of perhaps 1 cm (0.01 m). What would be the corresponding error in time measurement? Below is a table of possible heights along with their calculated times of flight.

Height (m)	Time of Flight (s)
0.15	0.350
0.20	0.404
0.25	0.452
0.30	0.495
0.35	0.535
0.40	0.571
0.45	0.606
0.50	0.639
0.55	0.670
0.60	0.700
1.067	0.934

At a height of about 30 cm notice that an error of 0.05 m corresponds to a time difference of about 0.040 s, so an error of 0.01 m would be roughly a time error of 0.008 s Since human reaction time is at least 0.1 seconds, time measurements of this accuracy are clearly impossible. The result obtained measuring height is more accurate than could be obtained using a stopwatch. Thus, measuring the height of the jump yields a better time value than directly measuring the time.

As a rule of thumb, length measurements are almost always better data than time measurements.

3.5 Hang Time in Basketball

The vertical jump is probably used as much in basketball as in any other sport. The center jump, rebounding, and the jump shot are important elements in any game of basketball. We shall re-examine the vertical jump, but this time we will consider some other aspects.

Let's consider the idea that basketball players hang in the air during a jump. One way to analyze this idea is to compare the times spent in different parts of the jump. To take a specific example, suppose that a player jumps 30 inches. Let's compare the time spent near the top of the jump with the time spent elsewhere in the jump.

First we calculate the time required to reach the top of the jump. To answer this question we use symmetry as we did before. We calculate the time to fall back down from a height of 30 inches:

$$\Delta x = 30 \text{ in} = 0.762 \text{ m}, \quad v_i = 0, \quad a = 9.80 \text{ m/s}^2$$

$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$
$$0.762 \text{ m} = 0 + \tfrac{1}{2}(9.8 \text{ m/s}^2)t^2$$
$$t^2 = 0.156 \text{ s}^2$$
$$t = 0.394 \text{ s}$$

A time of 0.394 s is required to either rise to the top of the jump or fall from the top of the jump to the bottom of the jump. Is the jumper near the top of the jump longer than she is near the bottom? Yes, as a matter of fact she is, and this contributes to the idea that she hangs in the air. By examining the equation used in the calculation above, we can discover the physical basis for this hang time effect.

Suppose we drop an object and let it fall. It has an initial velocity of zero, so our equation would simplify as follows:

$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$
$$\Delta x = 0 + \tfrac{1}{2}*g*t^2$$
$$\Delta x = \tfrac{1}{2}*g*t^2$$
$$\Delta x = (4.90)t^2$$

In one second (t = 1) an object falls 4.90 m, but in 2 seconds (t = 2) an object falls 19.6 m. If one doubles the time, then the distance fallen increases fourfold.

Another way to look at this situation is to observe that during s 2 sec fall, the falling object spends almost half the time in falling the first 4.90 m. During the remaining half of the time, however, the object covers a distance of 14.7 m. The object falls three times as far during the second half of the time period as it does during the first half of the time period.

This explains why basketball players hang at the top of their jump. They really do spend more time near the top than the bottom.

This situation is dramatized when we compare the time spent within 6 inches (0.152 m) of the top with the time spent within 6 inches of the bottom in the 30 inch jump. To fall 6 inches requires a simple re-calculation:

$$\Delta x = (4.90)t^2$$
$$(0.152 \text{ m}) = (4.90 \text{ m/s}^2)t^2$$
$$t = 0.176 \text{ sec}$$

To obtain the time spent within 6 inches of the bottom we calculate the time to fall 24 inches (0.610 m) and then subtract it from the time we obtained to fall the full 30 inches.

$$\Delta x = (4.90)t^2$$
$$(0.610 \text{ m}) = (4.90 \text{ m/s}^2)t^2$$
$$t = 0.353 \text{ sec for a 24 inch fall}$$
$$t = (0.394 \text{ s}) - (0.353 \text{ s}) \text{ for the last 6 inches}$$
$$t = 0.041 \text{ sec for the last 6 inches}$$

It took 0.176 sec for the first 6 inches of fall, but only 0.041 sec for the final 6 inches of fall. No wonder a viewer has the impression that Michael Jordan can hang in the air!

Problems to do:

1. Calculate the speed of a ball 5 sec after it is dropped. [Neglect air resistance!]

2. A baseball is thrown straight up with a velocity of +30 m/s. What is the velocity of the baseball after:
   A. 2 sec
   B. 4 sec

3. A punt rises for a time of 2 sec before falling. What was its initial upward velocity?

4. Three columns of numbers represent possible velocities of an object moving with constant acceleration. Which one set of numbers (A, B, or C) satisfies this condition?

v_A (m/s)	v_B (m/s)	v_C (m/s)	t (s)
0.0	0.0	0.0	0
10.0	20.0	30.0	1.0
30.0	40.0	50.0	2.0
60.0	60.0	60.0	3.0

5. Approximately how long must an object fall (without air resistance) to acquire a speed equal to the maximum speed of a world class sprinter?

6. An object has a constant acceleration of +3 m/s^2. Initially its velocity is +9 m/s. What is the velocity:
   A. after 3 s
   B. after 5 s

7. A runner is moving at 5 m/s at t = 2 s, and by t = 5 s moves at 11 m/s in the same direction. What is her average acceleration?

8. At a certain moment a sky diver's downward velocity is 14 m/s. Exactly 2.5 s later his downward velocity is 29 m/s. Calculate his average acceleration during this time interval.

9. Instead of 29 m/s, what would be the skydiver's velocity in #8 after 2.5 s if there had been no air resistance?

10. A volleyball player jumps to block a kill-shot. If we define "up" as positive for this problem, is the jumper's acceleration while rising positive or negative?

11. Calculate the average speed of a runner who requires 300 s to complete 1500 m.

12. A runner maintains a pace corresponding to 7 min/mi during a marathon race (26 mi 385 yd). How long did the run take?

13. Sprinters are said to reach maximum speed about 6 sec after a race begins. In this case would the fastest or slowest runner reach maximum speed closest to the finish line in a 200 m race?

14. If 6 s are required for a sprinter to accelerate from rest to a maximum speed of 9 m/s (see #13), then what is the sprinter's average acceleration? How far does she run during this period?

15. In basketball, a center jumps 30 in off the floor. How long will it take for the center to fall the first 5 in back down? How long for the first 10 in back down?

16. How long is a basketball player airborne who jumps 36 in straight up? Including both rising and falling motions, how long will he be within 9 inches of the top?

17. A trampoline artist is airborne for 2 s during a jump. What was her maximum height?

18. An object undergoes constant acceleration and its position is given below. What is its position at t = 6 s?

x (meters)	t (seconds)
0.0	0.0
10.0	2.0
40.0	4.0
? ?	6.0

19. Typically a skydiver stunts while falling at about 50 m/s. Normally a fall of 10-15 sec is needed to reach this speed. What time would be needed without air resistance.

20. A skydiver drops from the plane at t = 0 s. How far will she fall between t = 2 s and t = 3 s if we neglect air resistance?

21. A football kicked straight up is airborne for 4 s. At what speed does it strike the ground?

22. To what height will a football rise if it is airborne for 5 s?

23. A baseball travels straight up after being hit by the bat. A fan notices that it is airborne for 6 s. Neglecting air resistance, what was its initial velocity?

4. Starting a Race

Reaction Time

4.1 Reaction Time

We define "reaction time" to be the time interval between a stimulus and the response. The nature of the stimulus can affect the reaction time. If timers in a race react to the sight of the gun smoke from the starter's gun, they will start their stopwatches sooner than if they react to the sound of the gun. The nature of the response can also affect reaction time. A goalie reacting to a shot on goal with his glove will require less time than if he must react with his whole body.

4.2 Minimum Reaction Time

When working with athletes, the question arises as to whether a minimum reaction time might exist for a person. Dr. Samuel Williamson of New York University has been making measurements on magnetic fields surrounding the brain. He employs a variety of stimuli, and is able to detect changes in the magnetic field pattern surrounding the brain. He has correlated these changes with various stimulus-response situations, and in this way arrived at some interesting conclusions.

Magnetic Field Response Tests:

Type of Stimulus	Reaction Time (sec)
Visual (simple pattern)	0.080
Visual (complex pattern)	0.260
Motor stimulus	0.115

The minimum response time Williamson measured was 80 milliseconds (80 ms). Presumably this is somehow associated with the time required to process information in the brain. If this is so, a time of about 80 ms would be a lower limit for human reaction times. None of the available reaction data contradict this.

If Williamson requires that the subject press a button with a finger in response to a simple visual stimulus, the minimum response time increases to about 115 ms. As we shall see, this time is consistent with what we see in sports.

4.3 Starting a Race

In sprinting, swimming, and other races, a starter is employed and uses three commands: (1) on your mark, (2) get set, and (3) the gun. Athletes generally respond to the sound of the gun, whereas timers normally respond to the flash or puff of smoke given off. A false start occurs if a racer starts before the gun is fired, and results in disqualification of the athlete. This discourages anticipation and coerces athletes to react. Clearly a short reaction time is an advantage in a short race, but in longer races reaction time is not important.

In major competitions the athlete's reaction times, and possible false starts, are measured electronically. A device measures the force exerted by the feet of a sprinter on the starting blocks. The initial time is established when the gun is fired and the device can tell if the runner is pushing off too soon to earn a disqualification. Since the device can tell when the gun is fired and when the athlete pushes off of the block, it gives us another measure of reaction time. This device has been used to measure reaction times of athletes and records a typical number of around 0.12 seconds.

4.4 Goalie Reaction Times

In the mid-1970's **Sports Illustrated** published some reaction data for hockey goalies in the National Hockey League. Some of these numbers are reproduced in the table below. These reaction times are measured in the following way: the stimulus is a shot on goal from a pre-determined distance and the goalie is required to make a save in a specified manner. Then the distance and the puck velocity are used to determine the time. The minimum distance for which a save can be made for the specified save manner determines the reaction time for that type of save.

NHL Goalie Reaction Times

Goalie	Head	Leg	Stick	Total Body
Bower	0.20	0.46	0.27	0.56
Crozier	0.21	0.47	0.29	0.59
Hall	0.18	0.53	0.26	0.56
Sawchuch	0.19	0.51	0.27	0.63

Clearly a new element has entered. We now have a motion time. The goalie must move the stick, or all or some part of his body to make the save. As we might have expected the more massive the object to be moved, the larger is the resulting reaction time. This illustrates how various factors might enter into reaction time. Operationally, this is a very useful definition for the hockey goalie.

4.5 Reacting to Impulse Situations

Let's consider the impact between a ball and a racket, bat or club. Could a tennis player, baseball player, or golfer react to the impact quickly enough to correct a flaw in her swing? We know something about human reaction times, and such impact times have been estimated. For the golf club-golf ball collision, for instance, impact times are approximately 0.5 ms. In other impulse situations in sports impact times can be as large as 5 ms. Obviously these times are small compared with human reaction times. No corrections of swing flaws are possible by reacting to the impact.

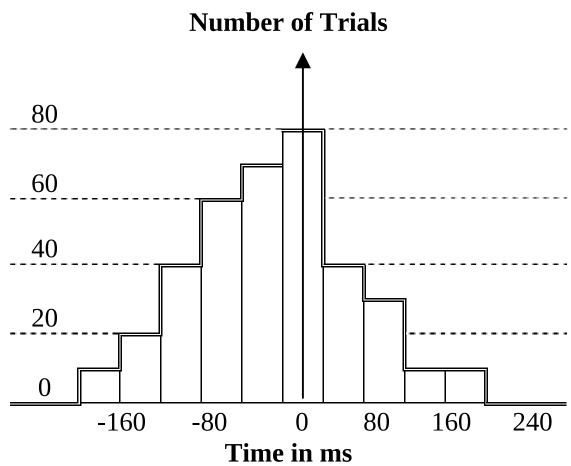
Let's do a numerical calculation for the case of a golf ball impact. Suppose the golfer has a reaction time of 0.15 s. The golf ball-clubhead impact time is about 0.0005 s, or $^{1}/_{300}$ of the reaction time. Long before the golfer reacts the ball has left the clubhead.

Another way to look at this is to ask where the ball is when the golfer has reacted. Suppose the ball velocity is 60 m/s. While the golfer is reacting to the impact the ball is traveling down the fairway and at that speed would travel about 9 m in 0.15 s.

4.6 Swimming Officials' Reaction Time

In the 1980's a new electronic timing system was installed for swim meets at Miami (Ohio) University. The timer is started by the gun and terminated by a touch pad in the pool. Times are measured to 0.1 ms and printed out to an accuracy of 1 ms. Back-up timers using stopwatches were employed during the transition period between the two timing systems, and both sets of data were used in a study to compare the reaction time of the manual timer to the actual time measured by the electronic timer.

Below is a graph (data rounded slightly) that shows the results for 385 separately timed events. Positive numbers mean the human timer was late, and negative numbers mean the human timer was early.



The spread of over a range of about 500 ms. Results are summarized below:

Anticipated	174	t_{AVG} = -0.069 s
Late	211	t_{AVG} = +0.064 s
Total	385	t_{AVG} = +0.004 s

From the results we can see that overall the stopwatch group did a great job of determining when the gun went off and when the swimmers touched the pad. Individually, however, the human timers were off by quite a bit.

4.7 Anticipation in Drag Racing

To start a drag race a bank of vertical lights is used. The lights are turned on in a definite regular cadence. The drag race driver watches the lights turn on and starts his car into motion when he anticipates the race will start. The regular cadence of the starting lights in drag racing is to be contrasted with the starting procedure used in a sprint or swimming event. The difference is in the nature of the stimulus. In a race the three commands do not come at precise intervals, and the timing of the commands varies with the starter.

Disqualification for false starts forces the sprinter or swimmer to react to the sound of the gun, whereas a drag racer must anticipate in the start.

4.8 Velocity-Distance Relation

For motion with constant acceleration we have seen that the velocity is proportional to the time. In fact, from one of our TNEOM:

$$v = v_i + a*t$$

we can see that **v** is proportional to **t** for constant **a**. In equal time intervals equal changes in velocity occur. For a freely falling body the data below shows this regularity.

t (sec)	v (m/s)	v (mi/hr)
0.0	0.00	0.00
1.0	9.80	21.95
2.0	19.60	43.89
3.0	29.40	65.84
4.0	39.20	87.79
5.0	49.00	109.73

Each one-second step in time produces an increase in the velocity (neglecting air resistance) of 9.80 m/s. A five-second fall give a speed on excess of 100 mi/hr. The relationship between velocity and distance is qute different. We can obtain insight into the velocity-distance relationship by considering another of the TNEOM:

$$v^2 = v_i^2 + 2*a*\Delta x$$

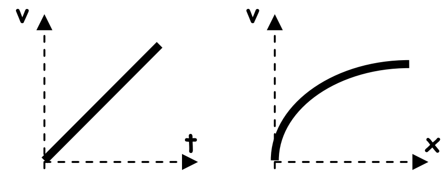
(or) $v^2 = 2*a*\Delta x$ if v_i is zero.

The $v_o = 0$ case is considered just for the sake of simplicity.

x (m)	v (m/s)	v (mi/hr)
0.0	0.00	0.00
10.0	14.01	31.34
20.0	19.81	44.32
30.0	24.26	54.27
40.0	28.01	62.66
50.0	31.32	70.07
60.0	34.31	76.76
70.0	37.06	82.91
80.0	39.62	88.64
90.0	42.02	94.00

We can see that the velocity and position are not proportional, but that doubling the velocity (such as 31.34 mi/hr to 62.66 mi/hr) corresponds to four times the distance (from 10 m to 40 m). Similarly, if the velocity triples (from 31.34 mi/hr to 94.00 mi/hr), then the distance increases nine times (from 10 m to 90 m).

The comparison between the **v-t** and **v-x** relations can be made graphically.



The **v-t** graph is a straight line, whereas the **v-x** graph is curved.

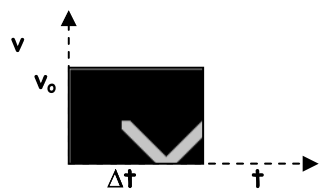
4.9 Area as Distance on a v-t Graph

The velocity vs. time graph is particularly useful because the slope is interpreted to be the acceleration. Now we learn how to interpret the area under a **v-t** graph. For simplicity we begin with the case for which the acceleration equals zero:

$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$

(or) $\Delta x = v_i*t$ if $a = 0$.

Let's consider a plot of this equation on the v-t graph below:

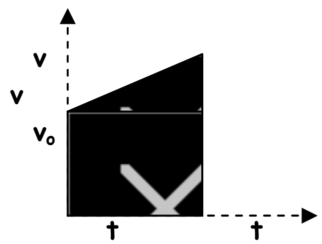


We see that a rectangle is defined having height v_i and width **t**. The area of this rectangle is the product of the base (**t**) times the height (v_i), or area $A = v_i*t$.

Evidently, at least for zero acceleration, both the distance **x** and the area **A** equal v_i*t. We have a new interpretation of the **v-t** graph. The area under the curve equals the distance covered.

Now we will show that this new interpretation is also valid when the acceleration is constant but not zero. Below is a second **v-t** graph for constant acceleration. For this case, we are plotting a graph of the TNEOM

$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$



To find a geometric expression for the area it is convenient to divide the area under the line into a rectangle and a triangle:

A_1 = base*height = $t*v_i$,
and A_2 = ½ base*height = $\frac{1}{2}\, t*(v-v_i)$.

Recall also the TNEOM that says

$$v = v_i + a*t$$

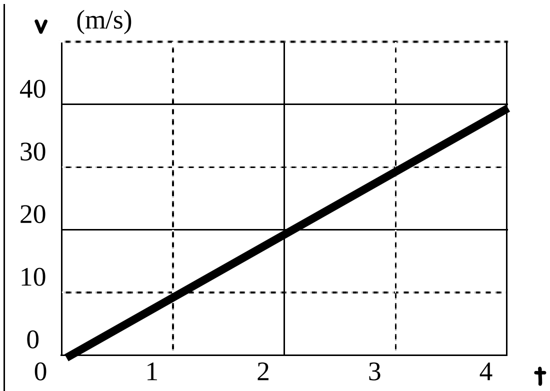
and this tells us that A_2 simplifies to $\frac{1}{2}*a*t^2$. Thus, for the entire area, we find that:

$$Area = v_i*t + \tfrac{1}{2}*a*t^2$$

This gives us the same relationship as before – the area under the **v-t** graph is the same as the distance covered by the object! This proof has been made for both the simple case of a = 0, as well as a more general case of a = constant. It turns out to be true for all accelerations.

Example – Distance from Area

Consider the graph below. This represents a heavy object like a sky diver in freefall (with no air resistance.) Lets find out the distance the sky diver has fallen during her first 4 seconds.



Let's calculate the area from the graph.

A = ½ base*height = ½ (4 s)*(40 m/s)
A = 80.0 m

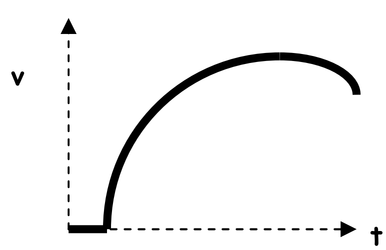
This tells us that the distance covered by the sky diver in the first 4 s is 80.0 meters. We can verify this by using the TNEOM where v_o = 0 (read from the graph) and a = 9.8 m/s2 (slope of the graph is gravity).

$$\Delta x = v_i*t + \tfrac{1}{2}*a*t^2$$
$$\Delta x = 0 + \tfrac{1}{2}*(9.8\ m/s2)(4\ s)^2$$
$$\Delta x = 80.0\ m$$

The agreement depends upon the accuracy with which one can read the graph. To three significant figures, the results agree!

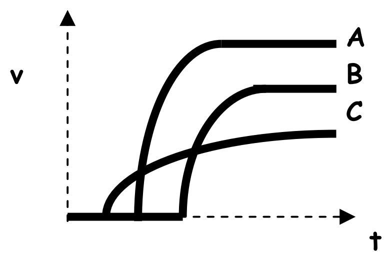
4.10 Problems to do:

1. The graph below shows velocity vs. time at the start of a sprint. The origin is when the gun goes off. Find the point on the graph (or part of the graph) corresponding to:
   A. maximum velocity
   B. maximum acceleration
   C. the reaction time of the runner

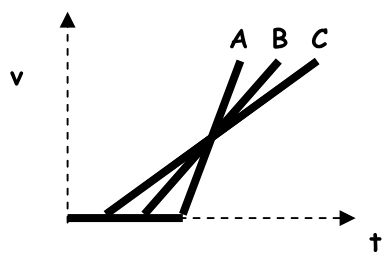


2. For the v-t graph in #1 draw three similar types of graphs. Make one of these have a shorter reaction time, a second have a larger maximum velocity, and the third have a larger maximum acceleration.

3. Three v-t graphs (**A**, **B**, **C**) are shown for swimmers. From the graph determine the swimmer with the :
   A. shortest reaction time
   B. largest maximum velocity
   C. largest maximum acceleration



4. Three v-t graphs (**A**, **B**, **C**) are shown for sprinters. From the graph determine the swimmer with the :
   A. shortest reaction time
   B. maximum acceleration



5. The first runner to move after hearing the gun has the smallest reaction time. Explain why the first runner to move need not be the same one:
   A. able to run the fastest
   B. able to develop the largest acceleration

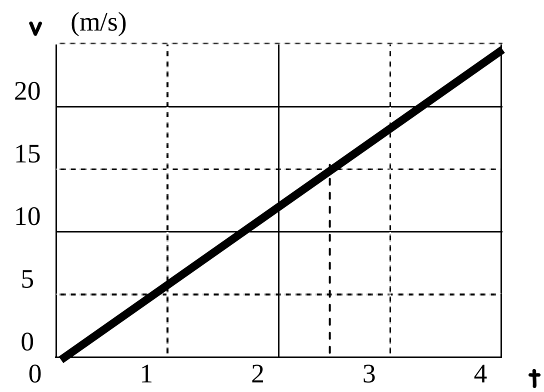
6. Explain why the reaction time defined by a sprinter coming off a block might be different than the reaction time determined by a meter stick drop experiment. Why would both of these numbers be larger than the 80 ms minimum measured by Williamson?

7. A standard "sucker bet" requires the victim to catch a dollar bill dropped through his fingers before it passes all the way through. What reaction time would be required to win he bet? Take the length of a dollar bill to be 15.4 cm, and assume the bill is halfway through the victim's fingers at the instant it is dropped.

8. A piece of paper is placed between nearly closed fingers and dropped. The measured reaction time is 100 ms. How far did the paper fall?

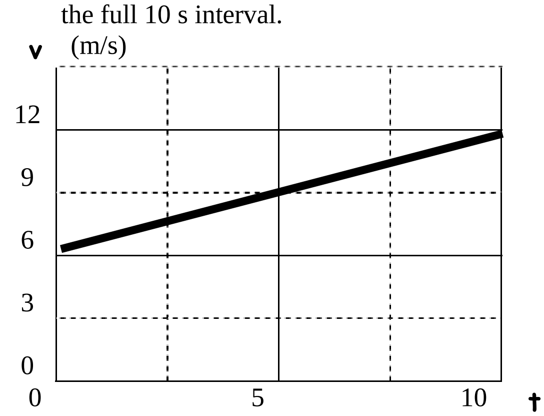
9. A baseball bat is dropped through the open hand of a player. Reacting to the sight of the falling bat the player grabs it after it has fallen for a time of 250 ms. What distance did the bat fall?

10. Using the graph below determine how far the object moves during the first 2.5 s. Solve the problem both by counting rectangles and by using the formula for the area of a triangle.

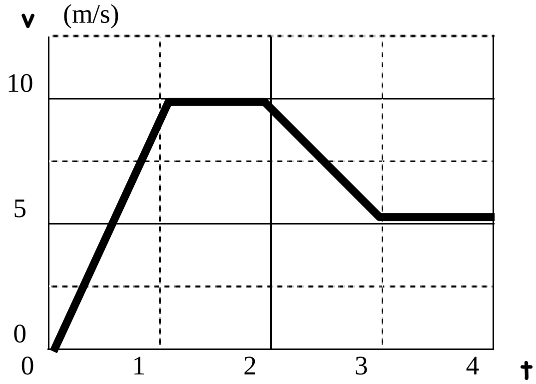


11. On the graph for #10 sketch two different graphs for which the object moves 25 m during the 2.5 s interval.

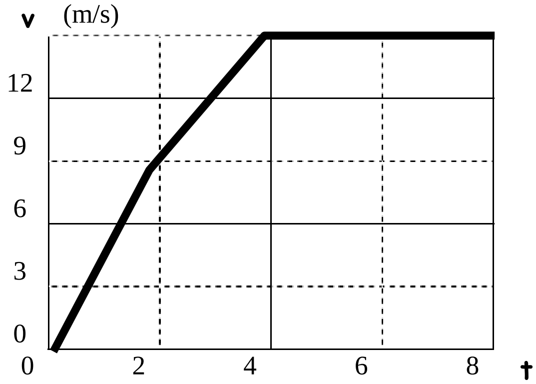
12. Determine the distance traveled by the runner whose **v-t** graph is shown. Consider the full 10 s interval.



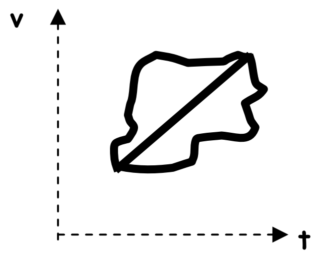
13. How far will a runner travel whose v-t graph is drawn below? Consider the whole 4 s interval.



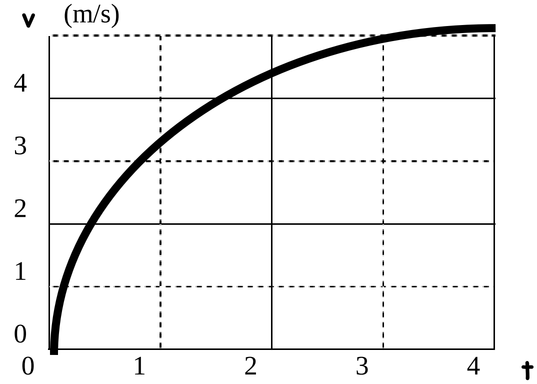
14. Velocity is plotted against time for a bicycle. How far does the bicycle move in 6 s?



15. Three **v-t** graphs are drawn connecting the sale pair of points. Which graph (top, middle, bottom) corresponds to the largest distance covered?



16. For the **v-t** graph shown determine the distance traveled during the first 3 s.



17. A sprinter comes to rest in accordance with the **v-t** graph shown. What distance is covered during the 12 s interval drawn?