1. Motion of an Arrow

Constant velocity motion in one direction

1.1 Position

Position specifies the location of an object. In a 100 meter race a runner is located somewhere between the start and the finish. We may give the position at any time relative to the start of the race. To specify the position the origin must be known. If a runner has traveled 42 meters along a track, we may say that her position is 42 meters. The symbol for position is x.

1.2 Displacement

The difference between two positions of an object is called displacement. Thus, if x_I is the initial position of a runner, and x_F is her final position, then

$$\Delta x = x_F - x_I$$

is the displacement of the runner between the initial time t_I and final time t_F. Displacement may be positive or negative and is measured in meters.

We use the symbol Δ (the Greek letter delta) to represent "change." This means that Δx is read as "change in x" or "delta x."

1.3 Time and Time Interval

The reading on a clock is the time. Thus, "ten minutes past noon" and "4.2 seconds after the gun fired" are both times. Time is measured in seconds, and the symbol for time is t.

In races employing a stopwatch, the starting time of the watch is often zero. However, the initial time need not be zero. In giving "lap times" we must subtract a starting time from a finishing time to obtain the time interval for the lap. We let t_I be the initial time and t_F be the final time so that:

$$\Delta t = t_F - t_I$$

is the time interval. Again, the Δ symbol is used to imply a "change in time."

1.4 Average Velocity

We are now ready to define our first derived quantity, average velocity, in terms of the concepts of position, displacement, time and time interval. During the time interval Δt, average velocity v_{AVG} can be determined by the following equation:

$$v_{AVG} = \Delta x / \Delta t$$

Average velocity is displacement over elapsed time, or the rate at which position was changed.

EXAMPLE – Average Velocity of an Arrow

Position-time data is in tabular form below. Let's determine v_{AVG} during (a) the first half second, (b) the full 4 seconds, and (c) the last half-second. (we'll also apply sig figs!)

x (meters)	t (seconds)
0.00	0.00
30.62	0.50
59.06	1.00
85.30	1.50
109.73	2.00
132.69	2.50
154.20	3.00
174.25	3.50
193.57	4.00

(a) From the table we read the values of position and time, and calculate Δx and Δt. For the first half-second we have:

Δx = 30.62 m – 0.00 m = 30.62 m
Δt = 0.50 sec – 0.00 s = 0.50 sec
v_{AVG} = 30.62 m / 0.50 sec = 61.2 m/s

(b) Similarly, for the full 4 seconds we have:
Δx = 193.57 m – 0.00 m = 193.57 m
Δt = 4.00 sec – 0.00 s = 4.00 sec

v_{AVG} = 193.57 m / 4.00 sec = 48.4 m/s

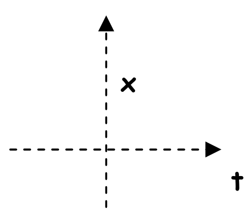
(c) Finally, for the last half-second we have:
Δx = 193.57 m – 174.25 m = 19.32 m
Δt = 4.00 sec – 3.50 s = 0.50 sec
v_{AVG} = 19.32 m / 0.50 sec = 38.6 m/s

We note several features of these results. The average velocity is not constant. It is 38.6 m/s near the end of the flight and 61.2 m/s near the beginning of the flight. Evidently, the arrow is slowing down. It is reasonable, therefore, that the average velocity for the whole flight, 48.4 m/s, lies between these extreme values.

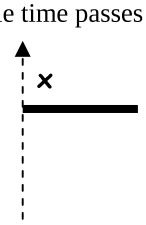
Data is often organized originally in the form of a table. The alternate representation of data is in the form of a graph. With a graph, patterns are noticed more quickly because they are more visual than a data table. (graphs are VIP- very important physics, especially on next test!)

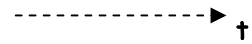
1.5 Position-Time Graph

We show position **x** and time **t** on a coordinate axis. On this graph, time is always on the x-axis (horizontal), and it is such that the future is to the right side while the past is on the left side. Far away from the origin (+) is up and far away from the origin (-) is down. Therefore, a point on this graph is a location along the line at a particular time. This graph is a continuous record of location and time.

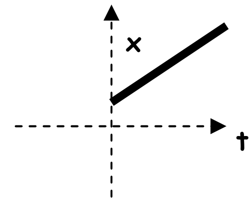


Some examples may help us interpret such a graph. Here is a graph where the position stays constant while time passes (the object is at rest):

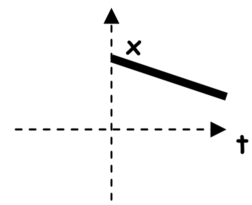




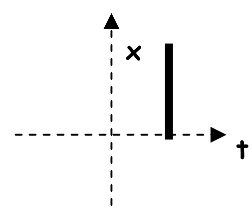
Here is a graph where the object moves away at constant speed (position increases at a regular rate) as time passes:



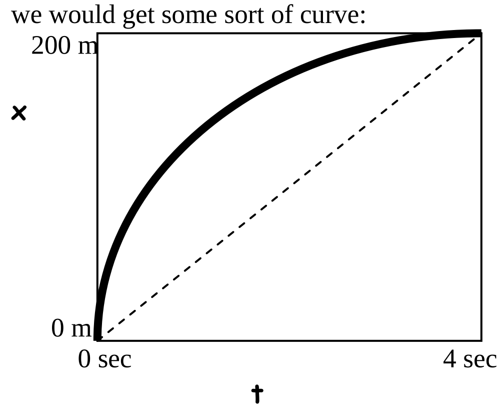
This graph shows an object moving toward the observar at a constant speed (position decreases at a regular rate) as time passes:



And this graph shows constant time but variable position. [This is most likely several objects at different locations at a certain time, but couldn't be a single object in motion.]



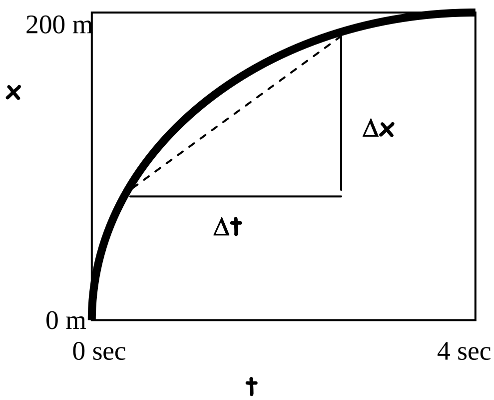
Let's return to the flight of the arrow from the table earlier. If we tried to graph the data points, we would get some sort of curve:



Okay, so the shape of the curve is exaggerated somewhat to prove a point. The actual data gives a curved shape (solid line), but if I just connect the start with the finish I get a straight line (dotted). The way we interpret the data may determine whether or not our final answer is a good one.

1.6 Average Velocity from the graph:

We need to select two points on the curve and find out the values for **Δx** and **Δt**.



What we see, interestingly enough, is that v_{AVG} on our graph **Δx/Δt** is exactly the same thing as the slope (rise/run). This is not an accident. It turns out that the slope of an **x-t** graph has a physical meaning – it is the average velocity of the object during that time interval!

x means location or position
Δx is displacement (change in position)
Δx = last position – first position

t = is a particular time
Δt is a time interval or elaspsed time
Δt = final time – start time

v_{AVG} – change in position / change in time

<u>To find v_{AVG}:</u>
Use the equation: $v_{AVG} = \Delta x/\Delta t$
Use x/t graph : v_{AVG} = rise/run

<u>Recognizing x-t graphs:</u>
Positive slope means moving forward
Flat (no) slope means not moving
negative slope means moving backwards

1.7 Brief Review (review in briefs?):

1.8 Problems to do:

Show work. Answers without steps don't count!

1. For the arrow data from before (1.4), give:
   A. its position at t_1 = 1.5 sec.

   B. its position at t_2= 3.0 sec.

   C. the displacement between t_1 and t_2 .

D. What is the time interval corresponding to this displacement?

2. For the same arrow data from before, give:

   A. its position at t_1 = 1.0 sec.

   B. its position at t_2= 3.5 sec.

   C. the displacement between t_1 and t_2 .

   D. What is the time interval corresponding to this displacement?

3. Calculate the average velocity for:

   A. the data in problem #1.

   B. the data in problem #2.

4. Explain what a negative velocity means.

5. Determine the average velocity for an object moving according to the position-time data given below. Use the time interval from 2 seconds to 4 seconds.

x (meters)	t (seconds)
0.0	0.0
2.7	1.0
21.6	2.0
72.9	3.0
172.8	4.0

6. For the data in #5, calculate the average velocity during each 1 second interval.

0-1 sec:

1-2 sec:

2-3 sec

3-4 sec:

7. Go back to the arrow data (p8). Make an **x-t** graph of the arrow and use the graph to estimate its position at a time t = 0.75 sec.

8. Graph the motion of the object given below.

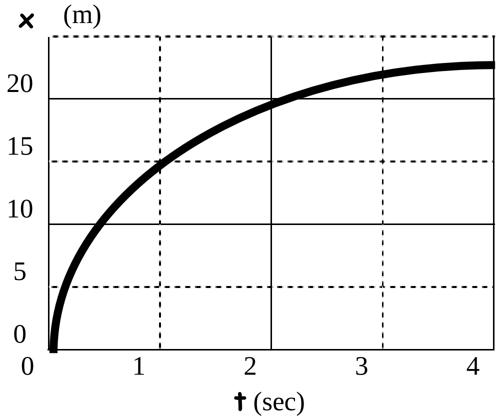
x (meters)	t (seconds)
0.0	0.0
1.0	1.0
4.0	2.0
9.0	3.0

Use the graph to estimate the position of the object at:

A. t = 0.5 sec

B. t = 2.75 sec

9. Graphically determine the slope of the curve below at **t** = 3.0 sec.



10. Using the data from question #8 determine the average velocity:

    A. from **t** = 0 sec to **t** = 1 sec.

    B. from **t** = 1 sec to **t** = 3 sec.

11. Plot the data from q#8 on an **x-t** graph, and draw a smooth curve connecting the points. From the graph, determine the average velocity between **t** = 0.5 s and **t** = 2.5 sec.

12. Sketch a graph of position vs. time for which:

    A. the velocity is always negative.

    B. the velocity is never zero.

    C. the velocity is always positive.

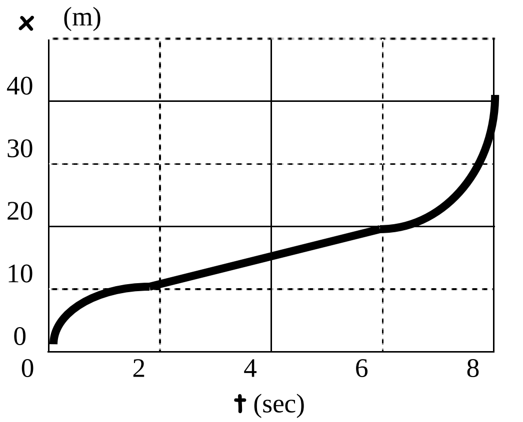
13. Using the graph from #9, determine the slope of the curve at:

    A. **t** = 0 sec

    B. **t** = 4 sec

14. Construct a **x-t** graph starting at **x** = 0 when **t** = 0, having the velocity 10 m/s at **t** = 1sec, and having the velocity 5 m/s at **t** = 2 sec.

15. The position of an object is shown plotted vs. time. During the interval from t = 2 s to t = 6 s, what is the average velocity of the object?



16. The position-time graphs are shown for runners A and B. For which runner is:
    A. the velocity constant.

    B. the velocity positive.

    C. the velocity negative at t =3 sec.

    D. the distance from the starting line greatest at t = 2 sec.

    E. the velocity zero at some time between t = 0 s and t = 4 sec.